Photon-momentum transfer in photoionization: From few photons to manySzczepan Chelkowski,^{1,*} André D. Bandrauk,¹ and Paul B. Corkum²¹*Laboratoire de Chimie Théorique, Faculté des Sciences, Université de Sherbrooke, Sherbrooke, Québec, Canada J1K 2R1*²*Joint Attosecond Science Laboratory of the National Research Council and the University of Ottawa,**100 Sussex Drive, Ottawa, Ontario, Canada K1A 0R6*

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In most models and theoretical calculations describing multiphoton ionization by infrared light, the dipole approximation is used. This is equivalent to setting the very small photon momentum to zero. Using numerical solutions of the (nondipole) three-dimensional time-dependent Schrödinger equation for one-electron (H-like) systems, we investigate the effect the photon-momentum transfer to the photoelectron in various regimes: from the few (one, two, three, eight) absorbed photons to multiphoton and tunneling regimes. We find that in all regimes the average electron momentum acquired from absorbed photons is a linear function of the average energy of the photoelectron, but the slope is different in the few-photon and multiphoton regimes. In the multiphoton regime, the photon-momentum signature in the photoelectron-momentum distributions (along the photon momentum) depends on the ellipticity of the laser polarization, with linear laser polarization spectra different from circular polarization spectra. For a given laser intensity the average electron-momentum gain from an absorbed photon is over two and a half times smaller for linear polarization than for circular polarization. However, for both polarizations, the average electron-momentum gain is nearly equal to the average electron energy divided by the speed of light.

DOI: [10.1103/PhysRevA.95.053402](https://doi.org/10.1103/PhysRevA.95.053402)**I. INTRODUCTION**

Early multiphoton ionization experiments using intense infrared pulses found the then amazing result that an ionizing electron often absorbs substantially more photons than the minimum needed for ionization [1]. This puzzling behavior led to the term above threshold ionization (ATI), a term still used today. The excess of absorbed photons also is related to high-order harmonic generation (HHG) discovered experimentally a few years later [2,3]. As neither phenomenon could be described by perturbation theory, these experiments stimulated the development of new theories. Several theoretical tools for modeling the ATI spectra have been developed, such as solving numerically the time-dependent Schrödinger equation (TDSE) [4] and strong-field approximation (SFA), often called the Keldysh-Faisal-Reiss (KFR) [5–7] theory, and for modeling the HHG spectra using the Lewenstein model [8]. In nearly all theoretical calculations the dipole approximation is used up to the laser intensity $I = 10^{17}$ W/cm² for the wavelengths λ in the 800–1064 nm range. This approximation means that the photon momentum and the laser magnetic field are both set to zero. Thus the dipole approximation is equivalent to the entire neglect of radiation pressure effects [9,10].

It has been widely believed (except Ref. [9]) that it is only necessary to go beyond the dipole approximation when the photoelectron reaches the relativistic energy regime. However, the crux of the problem is that the dipole approximation breaks down even in the case of long wavelengths, contrary to the textbook criterion: wavelength $\lambda \gg a_0$ (atom size). This criterion is therefore only valid in the perturbative one-photon regime. This breakdown of the dipole approximation is seen at relatively low laser intensities, $I \simeq 10^{14}$ W/cm² at which the photoelectrons are still nonrelativistic [9–14]. Already

in 1989 it was shown [9] that for infrared photons (e.g. $\lambda = 10$ μ m) significant radiation pressure effects appear in the tunneling regime, at the laser intensity $I = 10^{14}$ W/cm². Thus, since the radiation pressure effect is equal to U_p/c [11] where $U_p = (eE_0)^2/(4m_e\omega^2)$ is the ponderomotive energy and the well-known validity criterion of the tunneling model of the multiphoton ionization relies on the smallness of the Keldysh parameter $\gamma = \sqrt{I_p/2U_p}$ ($\gamma \ll 1$, I_p is the ionization potential of an atom), the tunneling model is only valid in a very narrow intensity-wavelength interval as shown in [10–12], and the model must be modified in the limit $\gamma \rightarrow 0$ when the radiation pressure effect grows as $U_p/c \sim 1/\gamma^2$. In [13] it has been demonstrated that the nondipole effects show up in the dynamics of the rescattering electron after tunneling at intensities as low as 10^{14} W/cm² for mid-IR ($\lambda = 2$ – 5 μ m).

Our previous [14,15] and current work is motivated by recent experiments [16,17] that confirm the breakdown of the dipole approximation in ATI momentum distributions at 800 nm where $\lambda \gg a_0$ at surprisingly low intensities, well below the relativistic regime expected to occur when the ponderomotive U_p approaches the electron's rest mass energy mc^2 . However, note that these experiments detect directly extremely small momenta shifts of the order of $p_z \sim U_p/c \sim 0.001$ a.u. due to extremely high resolution of the measurements [16,17]. Consequently, calculations based on a dipole approximation in which momentum distributions are averaged over the momentum p_z are still valid, i.e., the standard criterion $U_p \ll mc^2$ of the validity of dipole approximation is still relevant when the longitudinal momentum p_z (or the angle between electron momentum and the photon momentum) is averaged out. Specifically, these experiments show an asymmetric photoelectron-momentum distribution along the laser beam propagation direction (along the z axis): $S(p_x, p_y, -p_z) \neq S(p_x, p_y, p_z)$. We have shown that this asymmetry results from the radiation pressure exerted on the atom during photoionization. In the high-intensity

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long-wavelength limit, we found, for circularly polarized light, that

$$\langle p_z \rangle = (\langle E_{\text{el}} \rangle + 0.3I_p)/c, \quad (1)$$

where $\langle E_{\text{el}} \rangle$ is the average photoelectron energy. Using numerical solutions of two-dimensional (2D) TDSE [15], we confirmed the experimental findings [17] that the details of the longitudinal electron-momentum distributions (LEMD) [$f(p_z)$ obtained after integration of the total momentum spectrum $S(p_x, p_y, p_z)$ over p_x and over p_y] are very different for the linear from that for circular polarization. Our calculations also confirmed the experimental observation that a very narrow central cusp (present near $p_z \simeq 0$) in the $f(p_z)$ distribution has a counterintuitive shift in the opposite direction to the photon momentum. However, we found that the wings of LEMD $f(p_z)$ show the opposite shifts leading to overall average, positive, electron momentum $\langle p_z \rangle = (\langle E_{\text{el}} \rangle + 0.3I_p)/c$, i.e., the same overall effect for both linear and circular polarizations.

We also reported that one-photon absorption is different from the the strong-field, multiphoton absorption described by Eq. (1). The nondipole effect on the photoelectron in the case of single-photon absorption leads to

$$\langle p_z \rangle = (8/5)\langle E_{\text{el}} \rangle/c = (8/5)(\hbar\omega - I_p)/c. \quad (2)$$

This (8/5) factor may seem counterintuitive (since for high photon energies the photoelectron acquires more momentum than the photon disposes) but it was already reported in theoretical work in 1930s [18]. It has not been described in quantum mechanics textbooks except for a short comment and reference in [19] but has been known in astrophysics [20,21].

Our goal in the present paper is to investigate the transition region between multiphoton and one-photon ionization using numerical solutions of three-dimensional (3D) TDSEs for the H atom. We generalize the above linear rules (1) and (2) to the intermediate (low-intensity) multiphoton case and few-photon (two to three; ten or more in the case of the 800-nm laser) case as the more general form:

$$\langle p_z \rangle = (\alpha\langle E_{\text{el}} \rangle + \beta I_p)/c. \quad (3)$$

The appropriate values for the constants α and β are found from the numerical solutions of a TDSE. We also calculate the radiation pressure effect for the case of nine 800-nm photon (or more) ionization of a H atom, in an intermediate field strength regime ($I < 10^{14}$ W/cm²), in which the SFA approach used in our previous paper is invalid. Our goal is to investigate a gradual transition in the radiation pressure effect from the tunneling regime to the perturbative multiphoton regime. We also discuss the differences between circular and linear polarizations. In the case of the one-photon perturbative transition, the radiation pressure effect is the same for both polarizations. We find that at fixed laser intensity the influence of the radiation pressure from the linearly polarized laser beams is 2.5 weaker than from the circular polarization at the same laser intensity.

II. DESCRIPTION OF MODEL BASED ON TDSE

To solve numerically the three-dimensional (3D) TDSE describing the interaction of a H atom with an intense, linearly polarized laser pulse, we use the exact nonrelativistic

nondipole Hamiltonian (in the Coulomb gauge) in which the retardation, $t' = t - z/c$, is included in the vector potential $\vec{A}$. In atomic units ($m_e = \hbar = e = 1$) the 3D laser+system Hamiltonian (with regularized 3D Coulomb potential) is

$$\hat{H} = H_{\text{at}} + V_I, \quad H_{\text{at}} = -\frac{1}{2}\Delta - \frac{1}{\sqrt{r^2 + (1/16)^2}}, \quad (4)$$

where $r^2 = x^2 + y^2 + z^2$, Δ is the Laplacian in Cartesian coordinates, and the interaction potential for the laser propagating along z axis is

$$V_I(t - z/c) = -iA_x(t')\frac{\partial}{\partial x} - iA_y(t')\frac{\partial}{\partial y} + \frac{1}{2}[A_x(t')^2 + A_y(t')^2]. \quad (5)$$

We use the retarded laser vector potential $\vec{A}(t - z/c)$ with the sine-squared envelopes, i.e.,

$$A_x(t') = \frac{E_0}{\omega} \sin^2[\pi t'/t_p] \cos(\omega t'),$$

$$A_y(t') = \varepsilon \frac{E_0}{\omega} \sin^2[\pi t'/t_p] \sin(\omega t'), \quad (6)$$

where t_p is the total pulse duration, ε is the ellipticity, and the corresponding electric field is

$$E_x(t') = -\frac{\partial A_x}{\partial t}, \quad E_y(t') = -\frac{\partial A_y}{\partial t}.$$

We use the circularly polarized laser ($\varepsilon = 1$), except in Figs. 3, 9, and 10, where we compare results for the case of linear polarization ($\varepsilon = 0$).

The TDSE

$$i \frac{\partial \Psi(x, y, z, t)}{\partial t} = \hat{H} \Psi(x, y, z, t) \quad (7)$$

with the Hamiltonian (4) is solved numerically using the Fourier split-operator method [22] on a rectangular 3D grid of size

$$|x|, |y| < 384 \text{ a.u.} \gg \alpha_0 = E_0/\omega^2,$$

$$|z| < 256 \text{ a.u.}, \quad \Delta x = \Delta y = \Delta z = 0.25 \text{ a.u.},$$

i.e., 3072×3072 points are used in the x, y -plane grid and 2048 in the z -axis grid. The electron excursion amplitude α_0 , for $\lambda = 800$ nm at $I = 7 \times 10^{14}$ W/cm², is $\alpha_0 = 43.5$ a.u., i.e., it is much smaller than the grid size. This grid is large enough to minimize absorption on the grid edge, where an absorbing potential is introduced [23]. The system is initialized in the ground $1s$ state and the Schrödinger equation is integrated in time over the duration of the pulse. The time step used in the simulations is $\Delta t = 0.03$ a.u. = 0.72 as.

At the end of the pulse, $t = t_p$, we apply a mask function to the final wave function $\Psi(x, y, z, t_p)$ to remove the hydrogen bound states (and any low-energy continuum portions that are still located near the binding potential) and calculate the photoelectron momentum spectrum

$$S(\vec{p}) = |\tilde{\psi}(\vec{p})|^2, \quad (8)$$

where

$$\tilde{\psi}(\vec{p}) = (2\pi)^{-3/2} \int \exp(-i\vec{r} \cdot \vec{p}) \Psi_M(\vec{r}, t_p) d^3r \quad (9)$$

FIG. 1. Blue line with circles: expectation value $\langle p_z \rangle$ of the electron momentum p_z , calculated using the TDSE (7) as function of laser intensity. Black line with triangle: SFA-nondipole calculations. These are compared with the average electron values $\langle E_{el} \rangle$ (black circles: calculated using TDSE and red triangles obtained using nondipole SFA). Green lowest line shows the ponderomotive energy U_p . Circularly polarized, $\lambda = 800$ nm, $t_p = 4$ cycles, laser beam is used.

is the spatial Fourier transform of the masked wave function $\Psi_M(x, y, z, t_p)$ as in [24]. Next, we calculate the longitudinal electron-momentum distributions (i.e., in the laser propagation direction):

$$f(p_z) = \int \int S(\vec{p}) dp_x dp_y. \quad (10)$$

As a measure of radiative pressure we calculate the average value of the momentum component p_z as

$$\langle p_z \rangle = \frac{\int p_z f(p_z) dp_z}{\int f(p_z) dp_z}, \quad (11)$$

which we compare to the average electron kinetic energy

$$E_{el} = \frac{1}{2} \frac{\int S(\vec{p})(p_x^2 + p_y^2 + p_z^2) dp_x dp_y dp_z}{\int f(p_z) dp_z}. \quad (12)$$

III. NONDIPOLE (RADIATION PRESSURE) EFFECTS FOR THE NEAR-INFRARED LASER PULSES (MULTIPHOTON REGIME)

In Figs. 1 (lower intensity range) and 2 (higher intensity range) we plot the average momentum $\langle p_z \rangle$ transferred by circularly polarized infrared light ($\lambda = 800$ nm, $t_p = 4$ cycles) to the photoelectron (the blue line with circles) calculated using the TDSE (7), as a function of laser intensity I . For comparison, we also plot $\langle p_z \rangle$ calculated using the nondipole SFA described in [14] as a black line with triangles. For reference, we plot the corresponding electron average energy E_{el} and the ponderomotive energy U_p . We note that our TDSE results agree well with the SFA nondipole calculation and

FIG. 2. Same as Fig. 1, but for higher intensities, $I > 3 \times 10^{14}$. The lower part of the figure shows the radiation pressure effect for the linearly polarized beam.

confirm that at high laser intensities ($I > 3 \times 10^{14}$ W/cm 2) the gap $G = \langle p_z \rangle - E_{el}/c$, shown in Figs. 1 and 2, is equal to $0.3I_p/c$ as predicted by the relativistic tunneling model [25,26].

The origin of this gap is related to the nonzero initial momentum acquired during the electron under-barrier motion prior to the ionization step [25,26]. However, our TDSE calculations show that this gap decreases from $0.3I_p/c$ at high intensities to $G = 0.2I_p/c$ for $I = 5 \times 10^{13}$ W/cm 2 as shown in Fig. 3 which also shows the coefficients α and β defined via

Transition from S.F. to the multiphoton regime in H-atom at 800 nm

FIG. 3. Gap G , slope α , and parameter β .

FIG. 4. $f(p_z)$ longitudinal momentum distributions obtained using TDSE for linear and circular polarizations.

a linear fit defined in Eq. (3). We also note that the momentum $\langle p_z \rangle$ and the electron average energy $\langle E_{el} \rangle / c$ are significantly higher than U_p / c , whereas semiclassical models presented in [16,26] predict both equal to $U_p / c = (1/2c)(E_0/\omega)^2$ for the case of the circularly polarized laser beam. The slope α (i.e., the derivative of $\langle p_z \rangle$ with respect $\langle E_{el} \rangle / c$) increases from $\alpha = 1$ at high intensities to $\alpha = 1.2$ at lower intensity $I = 5 \times 10^{13}$ as it moves up to $\alpha = 1.6$ for a one-photon transition.

In Fig. 2 we also compare the radiative effect for circular polarization with that for linear polarization, in the higher intensity range than shown in Fig. 1. The momentum gain is 2.5 times weaker for linear polarization and the gap G between $\langle p_z \rangle$ and $\langle E_{el} \rangle / c$ is twice smaller than for circular polarization. In order to understand these differences in more detail we show next in Fig. 4 the longitudinal momentum distributions $f(p_z)$ for circular and linear polarization.

Figure 4 shows a much narrower distribution for linear polarization than for circular polarization as seen in experiments [27,28]. Our 3D calculation results show a much less sharp cusp than that in [15] where a 2D model and longer pulse ($\lambda = 3400$) were used. Since the radiative pressure effects are not seen in the scale used in Fig. 4 we narrow the momentum range in Fig. 5 to $|p_z| < 0.03$ a.u. This allows us to see the negative (counterintuitive) shift of the central narrow peak equal to -0.005 a.u., similar to the shifts reported in [15]. This counterintuitive negative shift seen in Fig. 5 originates from the combined Coulomb and magnetic force from the laser exerted on the recolliding electron, as demonstrated in [15,17].

Note that only the narrow central peak shows the negative shift. The outer part of the distribution $f(p_z)$ shows the shift in the opposite direction leading to the overall positive values of the average momentum $\langle p_z \rangle$. This change with respect to the dipole-approximation prediction is not a simple shift of the total distribution as it is for circular polarization.

FIG. 5. Same as Fig. 4, but for linear polarization and plotted in the very narrow p_z range.

To illustrate better this complex change in the distribution $f(p_z)$ for linear polarization, we show in Fig. 6 the normalized asymmetries

$$A(p_z) = \frac{f(p_z) - f(-p_z)}{f(p_z) + f(-p_z)}. \quad (13)$$

Our goal in Fig. 6 is to visualize with the help of $A(p_z)$ the complex change in the distribution $f(p_z)$ which is not clearly seen directly in the $f(p_z)$ shown in Figs. 4 and 5 which leads to the overall positive $\langle p_z \rangle$. For the linearly polarized pulse the counterintuitive asymmetry (corresponding to the cusp shift by -0.005 a.u.) only appears in the region of very small $|p_z| < 0.1$, i.e., there the asymmetry coefficient $A(p_z)$

FIG. 6. Asymmetry coefficient A corresponding to Fig. 4.

FIG. 7. Radiation pressure effect for the two-photon ionization, circular polarization.

is negative in a narrow interval $0 < p_z < 0.1$, which means that more electrons from this interval ionize preferentially in the counterintuitive (negative z) direction, whereas the positive, intuitive, overall momentum $\langle p_z \rangle$ originates from a wide interval of $p_z > 0.15$ in which the asymmetry coefficient $A(p_z)$ has positive values. We note that the asymmetry coefficient is positive in a much wider interval of p_z than the interval in which $A(p_z)$ is negative. Consequently, this leads to the overall positive radiative shifts seen in Fig. 2 for linear polarization. For comparison, we also show in Fig. 6 the asymmetry coefficients $A(p_z)$ for circularly polarized laser pulses for intensities $I = 2 \times 10^{14}$ and $I = 4 \times 10^{14}$ W/cm 2 . We observe a very different pattern for circular polarization where the asymmetry coefficient is always positive for $p_z > 0$, it grows linearly as function of p_z and is much larger than for linear polarization. At large p_z , $\simeq 0.5$ a.u., the asymmetry coefficient is equal to 0.13 at $I = 4 \times 10^{14}$ W/cm 2 . In summary, Fig. 6 shows very strong dependence of the radiation pressure effect on laser polarization.

IV. NONDIPOLE (RADIATION PRESSURE) EFFECTS FOR UV LASER PULSES: TWO- AND THREE-PHOTON IONIZATION

The radiative pressure effects for one-photon ionization corresponding to UV frequencies $\omega > I_p$ were discussed in [14], where we found that a simple relation $\langle p_z \rangle = (8/5)\langle E_{el} \rangle/c$ holds. Here, using TDSE, we are investigating an analogous relation for the case of two- and three-photon ionization, i.e., we are investigating the nondipole effects using TDSE for lasers having slightly smaller frequencies such that two-photon or three-photon transitions occur. Using circularly polarized UV pulses ($t_p = 10$ fs) in the frequency range $0.28 < \omega < 0.4$ a.u., we show in Fig. 7 the effect of radiation pressure for the case of two-photon transitions in the H atom. This frequency interval corresponds to the wavelengths

FIG. 8. Radiation pressure effect for the three-photon ionization, circular polarization.

$114 < \lambda < 163$ nm. Similarly, using the laser pulses in the frequency range $0.18 < \omega < 0.2$ a.u., we show in Fig. 8 the radiation pressure effect for the case of three-photon transitions. This frequency interval corresponds to the wavelengths $228 < \lambda < 253$ nm. In both cases we find that the rule similar to the one-photon ionization case holds: $\langle p_z \rangle = \alpha \langle E_{el} \rangle / c$, with $\beta = 0$. We find that for two-photon transitions $\alpha = 1.7$ whereas for three-photon transitions $\alpha = 1.6$, i.e., in both cases the slope α is close to the one-photon case where $\alpha = 1.6$. The fact that the slope α is slightly larger than 1.6 for the two-photon transitions is related to the fact that the second transition occurs directly to continuum from the $2p$ or $3d$ states [21] from which the one-photon radiative pressure effect is larger than from the $1s$ state, see Fig. 1 in [21]. However, when more than three photons are necessary to ionize the atom, we expect that the slope α decreases as a function of the number of photons absorbed. In the case of $\lambda = 800$ nm, when nine or more photons are absorbed, displayed in Figs. 1–3, this slope is $\simeq 1.2$ at low intensity, $I = 5 \times 10^{13}$ W/cm 2 , whereas it is equal to 1 at high intensities, see Fig. 2. Moreover, in this nine-photon case the coefficient β is equal to 0.3 at $I = 5 \times 10^{13}$ W/cm 2 , i.e., both coefficients α and β approaching the tunneling case, Eq. (1).

So far, we have investigated in Figs. 7 and 8 the effect of radiation pressure on two- and three-photon transitions, using circularly polarized laser pulses. Figures 9 and 10 are similar to Figs. 7 and 8 but they show the case of linear polarization. For comparison, we also plot in these figures the longitudinal average momenta $\langle p_z \rangle$, Eq. (11), for circularly polarized laser pulses. We recall that in the case of perturbative one-photon absorption the radiation pressure is the same for both linear and circular polarizations, while for two-photon absorption, shown in Fig. 9, there is small difference between $\langle p_z \rangle$ for linear and circular polarizations. For the three-photon process, Fig. 10, the difference between the two polarizations is surprisingly large. Clearly this is a signature of the emergence

FIG. 9. Same as Fig. 7, but for the linearly polarized laser. The solid line shows $\langle p_z \rangle$ for circular polarization displayed in Fig. 7.

of the post-ionization electron-ion interaction that will develop into full recollision. We find that $\langle p_z \rangle$ is nearly three times smaller for linear polarization than for circular polarization and even twice smaller than the value of $\langle E_{el}/c \rangle$. Note that in all cases that we have studied here, and in our previous work [14,15], $\langle p_z \rangle$ is larger than $\langle E_{el}/c \rangle$. If we use Eq. (3) for $\langle p_z \rangle$ as shown in Fig. 10 (linear polarization) we conclude that the coefficients α and β depend strongly on the laser frequency. For low frequency when a nonresonant transition occurs ($\omega < 0.19$ a.u.) we obtain $\alpha \simeq 1$ but a negative value of β near the frequency $\omega \simeq 0.18$. Unfortunately, this simple fitting procedure is not possible at higher frequencies, for

FIG. 10. Same as Fig. 8, but for the linearly polarized laser. The solid line shows $\langle p_z \rangle$ for circular polarization displayed in Fig. 8.

linear polarization (three-photon transitions), shown in Fig. 10. Otherwise, Eq. (3) works very well for all other cases that we have investigated here, in particular for circular polarization.

We believe that large difference between the linear and circular polarization seen are related to the different selection rules (for linear and circular polarizations) allowing in the linear polarization case to populate more bound intermediate states than in the circular case, e.g., in the second photon absorption of a circularly polarized photon only the $l = 2, m = 2$ state is populated; whereas in linear polarization, two states, $l = 2$ and $l = 0$, are populated. This creates larger Stark shifts for linear polarization which causes the nonlinear dependence of $\langle p_z \rangle$ on $3\omega - I_p$. We think that higher-order, higher than 3, perturbative calculation would explain these effects.

Another explanation of the unusual behavior in the case of three-photon ionization seen in Fig. 10 is related to the interaction term $\vec{A}^2$, which for circular polarization (near the peak of the envelope) is $\vec{A}_{\text{circ}}^2 = (E_0/\omega)^2/2$ whereas for linear polarization $\vec{A}_{\text{lin}}^2 = (E_0/\omega)^2 \cos^2(\omega t') = (E_0/\omega)^2 [1/2 + \cos(2\omega t')]$, i.e., for linear polarization the interaction contains the oscillatory term $\cos[2\omega(t - z/c)]$. Thus for circular polarization, the term $\vec{A}^2$ is irrelevant in the lowest order of the perturbative calculations (because of the absence of the 2ω oscillatory contribution), whereas for linear polarization the final perturbative transition amplitude is a result of coherent superposition of two terms present in the interaction, $\vec{A}^2/2$ and $\vec{A} \cdot \vec{p}$, leading to the possibility of cancellations and reduction of the resulting $\langle p_z \rangle$ value.

V. CONCLUDING REMARKS

In conclusion, we have studied the transfer of laser photon momentum to an atomic electron in various regimes. We investigated in detail the change in this transfer in the intermediate regime between single-photon ionization and strong field multiphoton ionization, identifying the sensitivity of the longitudinal momentum $\langle p_z \rangle$ gain by the electron to the order of nonlinearity, i.e., to the number of photons absorbed. The aim of previous work [16,28] was to observe the transition from perturbative to nonperturbative behavior through its signature in the p_z momentum distribution. Our TDSE numerical simulations which lead us to a simple linear relation (3) bear on this issue. The results indicate that the order of field-induced nonlinearities (i.e., number of photons absorbed) influence observables such as radiation pressure effects that are experimentally accessible [16,17]. For all the cases (except three-photon absorption for linear polarization), we see a simple linear dependence of $\langle p_z \rangle$ on the photoelectron energy E_{el} , $\langle p_z \rangle = \alpha \langle E_{el} \rangle / c + \beta$, with slopes α ranging from 1 (multiphoton strong field regime) to $\alpha = 1.7$ (three-photon, circular polarization) and $\alpha = 1.6$ for one and two-photon transitions. These high values of α originate from the *gradient* in the momentum probability distribution in the atom's initial state which for the $1s$ state falls as $[1 + p^2]^{-n}$ leading to $\alpha = 2n/5$ with $n = 4$ for the $1s$ state, i.e., the momentum transfer is proportional to the power " $n/2$ " in the initial $1s$ wave function for one-photon transitions from a $1s$ state. This explains the origin of the surprising $(8/5)$ factor in the case of one-photon processes. Additional shifts, $\beta = 0.3$, in Eq. (3),

were found in the multiphoton strong-field regime, while $\beta = 0$ applies to the one-, two-, and three-photon absorption regimes.

Finally, we suggest that radiation pressure effects should be relevant to complex photoelectron-momentum distributions obtained with bichromatic circularly polarized pulses which lead to circularly polarized HHG [29] as well as to counter-propagating pulses which separate periodically electric and

magnetic field effects in the nondipole regime [30] allowing cancellation of the magnetic force exerted on the electron.

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