DEVELOPMENT OF A FREQUENCY STABILIZED 422-nm DIODE LASER
SYSTEM AND ITS APPLICATION TO A $^{88}$Sr$^+$ SINGLE ION OPTICAL
FREQUENCY STANDARD

ANDREW SHINER

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2. Dr. William vanWijngaarden
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“Now the general who wins a battle makes many calculations in his temple ere
the battle is fought. The general who loses a battle makes but few calculations
beforehand. Thus do many calculations lead to victory, and few calculations to
defeat.”

The Art of War, Sun Tsu ~515 BCE
Abstract

A frequency standard based on a laser source probing an ultra narrow optical transition in a single trapped and laser cooled $^{88}\text{Sr}^+$ ion is under development at the National Research Council of Canada. This thesis consists of the development of a new frequency-stabilized diode laser system at 422-nm which has been used for Doppler laser cooling of the ion as well as fluorescence detection of the ion’s electronic state. The linewidth of this source has been narrowed to 2MHz and its absolute frequency has remained stabilized to an atomic reference transition for periods exceeding 12 hours. Results using this new source are presented demonstrating a significant reduction in ion temperature enabling the linewidth of its ultranarrow transition to be resolved at the Hz level. Measurements of the absolute optical transition frequencies between hyperfine components of the $5s^2S_{1/2} - 6p^2P_{1/2}$ transition in both $^{85}\text{Rb}$ and $^{87}\text{Rb}$ at 422-nm are also presented.
Acknowledgements

My relationship with the NRC Frequency and Time group began as a summer following the third year of my undergraduate degree. In considering the acknowledgements section for this thesis, I am struck by how much of what I know can be directly traced to things that the members of this group have taught me. While it occurs to me that it will not be possible to adequately thank these people for all that they have done, I will at least mention them here. First and foremost I wish to thank my supervisor Dr. Alan Madej. As a full time scientist at the NRC, his decision to seek out funding for, and then to supervise graduate students has gone well beyond his job description. I have benefited tremendously from his experimental skill, depth of knowledge and generosity. The example he has set through the care with which he conducts research and the way that he interacts with the scientific community will continue to guide me in the future. I would also like to thank Dr. Pierre Dubé whose door was always open, and through which I walked frequently during this work. Pierre’s enthusiasm for experimental work coupled with his wide ranging knowledge and willingness to help has been invaluable throughout this thesis. Both Alan and Pierre were directly involved in the ion trap experiments presented in Chapter 5. I would like to acknowledge the contribution by Dr. John Bernard who operated the optical frequency comb which was used to make absolute optical frequency measurements of the Rb lines presented in this work. John has
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Prior to moving to Ottawa to conduct the research component of my thesis at the NRC, I spent 8 months at York University completing the coursework requirements for this degree. My time at York was made much more enjoyable by the cordial welcome and frequent advice from my co-supervisor Dr. William van Wijngaarden. William has been a terrific resource while I was preparing to defend this work. His efficient organization of the committee for this thesis and his assistance in dealing with the York administration has been invaluable.

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1 Introduction

The years following the invention of the laser in 1960 have seen a revolution in atomic and molecular spectroscopy. It is now possible to laser cool a single trapped ion to a temperature where motional broadening of its atomic transitions is no longer observed. A laser cooled ion represents a nearly perfectly isolated quantum system, which while trapped, can be interrogated for indefinitely long periods of time [1]. Advances in laser stabilization with active frequency control have enabled the development of probe laser systems that exhibit spectral linewidths on the Hz level with an associated $Q = \Delta \nu/\nu$ of better than $10^{14}$ [2]. Such a light source can resolve an ultra narrow transition in a trapped and laser cooled ion and, when stabilized onto such a transition, have demonstrated frequency stabilities of 1 part in $10^{15}$ in 100 seconds [3]. Through the use of an optical frequency comb [4], the absolute optical frequency of a probe laser when stabilized onto an ultranarrow atomic transition can be directly compared to other optical frequencies or to the microwave frequency of the Cs atomic clocks that define the SI second. Measurements have now been made of long lived transitions in trapped and laser cooled ions with a level of stability that was limited by the stability of the best Cs atomic clocks [5]. This situation is analogous to 1968 when fluctuation in astronomical time observed by spectroscopists motivated the redefinition of the SI second in terms of the frequency of a microwave transition between two hyperfine levels in a caesium atom [6].

At the National Research Council of Canada’s Institute for National Measurement Standards,
a next generation frequency standard is under development based on probing an ultranarrow optical ‘clock’ transition in a single trapped and laser cooled $^{88}$Sr$^+$ ion. The major thrust of this thesis was the development of a new laser source at 422nm that is now used to excite resonance fluorescence and hence cool the $^{88}$Sr$^+$ ion in the NRC ion trap experiment. This thesis begins with some elements giving a quantum mechanical description of the $^{88}$Sr$^+$ ion, its spectra, and outlines some of its properties that are important for its development as a time/frequency standard. The following chapter describes the experimental methods that are used in order to realize a frequency standard based on probing a trapped and laser cooled $^{88}$Sr$^+$ ion. Chapter 4 describes in detail the development of a new cooling laser source based on a GaN laser diode operating at 422nm. Work is described showing the methods used to narrow the laser linewidth using a Fabry-Perot resonant cavity. In addition, the techniques used to stabilize the absolute frequency of the laser onto a saturated absorption line in $^{85}$Rb are described. Chapter 5 gives the results from several studies which demonstrate the improved performance of the $^{88}$Sr$^+$ based frequency standard that was achieved using this new cooling laser source. The thesis concludes with a chapter describing the results of absolute measurements of the optical transition frequencies in Rb near 422nm including the line used to stabilize the cooling laser for the $^{88}$Sr$^+$ ion trap experiment.
2 The $^{88}\text{Sr}^+$ Ion

2.1 Introduction

There exists a number of essential requirements for atomic ion absorbers to be used as optical frequency references. The ion needs to have a long lived transition between the ground state and a metastable excited state so that the natural linewidth is as narrow as possible. A frequency standard can be defined by measuring the transition frequency of this long lived ‘clock’ transition provided that the ion is sufficiently isolated from the surrounding environment. To reduce broadening and frequency shifts of the clock transition resulting from the ion’s motion, it is first laser cooled prior to attempting to excite the clock transition. To facilitate efficient laser cooling, an ion must have a strongly allowed transition that will allow the ion to rapidly absorb and reemit the cooling radiation. In addition, the clock transition must have a low sensitivity to systematic shifts such as the second order Zeeman shift and the Stark shift. Also, from a practical standpoint, the selected ion must have these transitions at frequencies that can be accessed with readily available laser sources. The Optical Frequency Standards group at the NRC is developing a trapped and laser cooled $^{88}\text{Sr}^+$ ion as a new optical frequency standard. To date, this ion has been accepted as both a secondary realization of the meter [7] as well as a secondary realization of the second [8].

The Sr ion consists of a filled [Kr] shell plus one valence electron. As with other ions of the
group IIa, Alkaline Earth Metals, Sr\(^+\) has metastable \((n-1)d\) states and a short lived \(np\) level as its first two excited doublet states. The exceptions to this from group IIa are Mg\(^+\) and Be\(^+\) whose principle quantum numbers are not high enough to access a metastable \(d\) level in the first excited state. The \(^{88}\text{Sr}\(^+\)) energy levels that are used for the realization of this frequency standard as well as the sensitivities of this ion to some shifts such as the those resulting from magnetic fields will be described in the following sections.

### 2.2 \(^{88}\text{Sr}\(^+\)) Energy Levels

A partial level diagram for \(^{88}\text{Sr}\(^+\)) is shown in Figure 2.1. The ground state for \(^{88}\text{Sr}\(^+\)) is \(5s^2S_{1/2}\). The ion is detected and laser cooled by excitation on the \(5s^2S_{1/2} - 5p^2P_{1/2}\) resonant transition at 422nm. This transition is short lived (7ns) and typically scatters > \(10^7\) photons per second allowing for straightforward detection of the ion’s fluorescence using photon counting detection.

In addition to the ground state, the \(5p^2P_{1/2}\) level can decay with 1:13 branching ratio to the long-lived \(4d^2D_{3/2}\) level. To pump the ion out of this energy level, a laser at 1092nm is also applied while cooling the ion. The reference ‘clock’ transition is the \(5s^2S_{1/2} - 4d^2D_{5/2}\) transition at 674nm (445 THz). This transition is electric quadrupole allowed and in the presence of a dc magnetic field, the transition is split into 10 Zeeman components. The clock transition has a very long natural lifetime yielding a linewidth of 0.4 Hz giving a \(Q = f/\Delta f\) of \(10^{15}\).

![Figure 2.1: Partial energy level diagram for \(^{88}\text{Sr}\(^+\)).](image)
The long lifetime of this transition makes direct detection of the absorption of 674nm photons impractical. Instead, the clock transition is observed from that fact that while in the $4d^2D_{5/2}$ state, the ion is no longer able to undergo scattering on the S-P 422 nm transition. The absence of fluorescence photons at 422nm during the cooling cycle indicates that the ion is shelved in the $4d^2D_{5/2}$ state having undergone a clock transition. This quantum jump technique allows for nearly 100% efficiency in detecting these extremely weak transitions and is described in greater detail in Section 3.4.

2.3 Zeeman Splitting of the Clock Transition

While confined inside the ion trap, the ion is exposed to a $\sim 14\mu T$ magnetic field which defines the ion’s quantization axis. This field is normally arranged to be nearly parallel with the probe laser although the facility exists with this trap to have the quantization axis point in any chosen direction or to suppress the background magnetic field altogether. The $^{88}\text{Sr}^+$ electrons interact with the magnetic field through the Zeeman effect which results in the lifting of the degeneracy in the $M_j^\prime$ ground state and $M_j^\prime\prime$ excited state. For the present case of a small magnetic field, the energy shift due to the Zeeman effect is small and can be treated as a perturbation in the LS basis. This shift is given by [9]:

$$h\Delta \nu = \mu_B B (g_j^\prime M_j^\prime - g_j^\prime\prime M_j^\prime\prime). \quad (2.1)$$

Where $h$ is Planck’s constant, $\Delta \nu$ is the shift in the transition frequency, $\mu_B$ is the Bohr magneton, $B$ is the magnetic field strength, $M_j$ is the level’s magnetic quantum number and $g_j$ is the Landé g-value defined below [9]:
\[ g_j = g_l \frac{J(J+1) + L(L+1) - S(S+1)}{2J(J+1)} + g_s \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}. \]  

(2.2)

In the above equation, \( L \) and \( S \) are the total orbital and spin quantum numbers respectively. The \( g \)-factors are \( g_s \approx 2 \) for electron spin and \( g_l \) for orbital angular momenta. The value for \( g_l \) is typically given as \( g_l \equiv 1 \) however closer agreement between the calculated and experimentally realized Zeeman splitting was achieved by making precision measurements of the transition frequency for each component at a given magnetic field strength, and then fitting (2.1) and (2.2) with \( g_l \) as a free parameter. The resulting value of \( g_l = 0.999902 \) has shown much better agreement between experiment and theory for the range of magnetic field values used in this experiment\(^1\). The calculated Landé \( g \)-factors are as follows:

\[
\begin{align*}
g_{5/2} &= 1.200 \, 385 \\
g_{1/2} &= 2.002 \, 319
\end{align*}
\]

The calculated Zeeman splitting between the various sublevels of the S-D ‘clock’ transition are summarized in Table 2.1.

### 2.4 Relative Intensities of the Electronic Transitions:

Much of the work in this thesis involved efforts to pump the ion between specific pairs of Zeeman levels of the ion’s ground and excited states. The relative strengths of transitions between different pairs of Zeeman components depends on the spatial coordinates of the electron undergoing the transition as well as the polarization state of the field inducing the

\(^1\)The value for \( g_l \) was determined by the NRC \(^{88}\text{Sr}^+\) ion trapping group in work that preceded this thesis
An electromagnetic field interacting with an atom can be described in terms of its vector $A$ and a scalar $\phi$ potentials as follows:

$$E = -\nabla \phi - \frac{\partial A}{\partial t} \quad (2.3)$$

$$B = \nabla \times A. \quad (2.4)$$

The laser field is approximated as a plane wave propagating in a direction given by $k$ with angular frequency $\omega$. The vector potential for this field is given by:

$$A = A_0^* e^{i(\omega t - k \cdot r)} + A_0 e^{-i(\omega t - k \cdot r)}. \quad (2.5)$$
For sufficiently low field intensities, we can treat the field as a perturbation of the ion’s unperturbed hamiltonian \( H_o \). The hamiltonian for the ion plus the perturbation is:

\[
H = H_o + H'
\]  

(2.6)

The perturbing Hamiltonian describes the interaction energy between a single electron charge, \(-e\) and the applied external radiation field, and is given as [9]:

\[
H' = -\frac{e}{m} \mathbf{A} \cdot \mathbf{p} + \frac{e^2}{2m} \mathbf{A}^2.
\]  

(2.7)

For weak laser fields, the term that is second order in \( \mathbf{A} \) is negligible compared to the first order term. When solved using perturbation theory and the Rotating Wave Approximation [9], it is found that the dependence of the transition rate on the coordinates of the ionic electron is given by the square of the matrix element connecting the (unperturbed) stationary states \( |\psi_1\rangle \) and \( |\psi_2\rangle \) as:

\[
|\langle \psi_2 | \frac{e}{m} \hat{\mathbf{e}} \cdot \mathbf{p} e^{i(\mathbf{k} \cdot \mathbf{r})} |\psi_1\rangle|^2.
\]  

(2.8)

This transition probability depends on the polarization direction \( \hat{\mathbf{e}} \) as well as the spatial variation of the electromagnetic field through the term \( e^{i(\mathbf{k} \cdot \mathbf{r})} \). Because the wavelength of the EM field \( \lambda \) is much larger than the ionic dimension \( \frac{a_0}{Z} \) at optical wavelengths, the term \( e^{i(\mathbf{k} \cdot \mathbf{r})} \) can be expanded in powers of \( \mathbf{k} \cdot \mathbf{r} \). The lowest order term in this expansion completely dominates (2.8) provided that this matrix element when evaluated with the lowest order term is non zero. This first order term in the multipole expansion describes dipole transitions which
are typically a factor of \(10^8\) more probable than the quadrupole transitions arising from the second order multipole term [10].

The relative strengths of electric dipole transitions between different Zeeman sublevels is given by the matrix element of the electric dipole operator \(p = \hat{e} \cdot r\) between the ground state \(|\psi_1\rangle\) and the excited state \(|\psi_2\rangle\) as:

\[
|\langle \psi_2 | \hat{e} \cdot r | \psi_1 \rangle|^2
\] (2.9)

The second order term in this multipole expansion leads to magnetic dipole as well as electric quadrupole transitions. The calculation of these transition probabilities has been presented in several texts [9–11], most of which briefly introduce quadrupole transitions following a detailed discussion on electric dipole transitions. A thorough discussion of multipole radiation is given in [12], the treatment of which goes beyond the scope of this thesis.

Similarly, the relative intensities for quadrupole transitions are found from the matrix element of the electric quadrupole moment operator \(Q\). This operator which can be derived from the operator in (2.8) can also be found as the quantum mechanical analog of the energy due to a distribution of classical electric quadrupoles [13] interacting with an electromagnetic field:

\[
H_Q = \sum_{ij} \sum_{\alpha} \frac{1}{2} q_\alpha (r_{\alpha i} r_{\alpha j} - \frac{1}{3} \delta_{ij} r_{\alpha}^2) \frac{\partial}{\partial x_i} E_j(0)
\] (2.10)

where \(i, j = x, y, z\) and \(\pm q_\alpha\) is the charge of each element comprising the quadrupole, \(\alpha\) labels each charge comprising the quadrupole and \(E(0)\) is the electric field at the origin. In analogy to this, the quadrupole matrix element is:
\[\langle \psi_2 | -\frac{e}{2} \sum_{ij} (r_{\alpha i} r_{\alpha j} - \frac{1}{3} \delta_{ij} r_{\alpha}^2) \frac{\partial}{\partial x_i} E_j(0) | \psi_1 \rangle \]  

(2.11)

where \( | \psi_1 \rangle \) and \( | \psi_2 \rangle \) are the ground and excited states respectively.

### 2.4.1 Relative Transition Strengths of Clock Transition Between Different \( M_J \) sublevels

The \( 5s^2S_{1/2} - 4p^2D_{5/2} \) clock transition consists of two ground state sublevels and six excited state sublevels which have a \( \sim 100 \text{kHz} \) splitting in the \( \sim 14 \mu \text{T} \) magnetic field typically present in the ion trap. The probe laser can be tuned to excite transitions between any pair of ground and excited state sublevels provided that the quadrupole selection rule \( \Delta m = 0, \pm 1, \pm 2 \) is satisfied.

As was described in the beginning of this section, the relative strengths of transitions between different pairs of Zeeman components depends on the orientation of the quantization axis relative to the direction of the probe laser as well as the probe laser’s polarization state.

The relative strengths of transitions between different \( M_J \) sublevels of the clock transition depends on magnitude squared of the expectation value of the quadrupole matrix element (2.11) when connecting the ground and excited states. As was shown in [13] this can be factored as follows:

\[\langle \psi_2 | -\frac{e}{2} \sum_{ij} (r_{\alpha i} r_{\alpha j} - \frac{1}{3} \delta_{ij} r_{\alpha}^2) \frac{\partial}{\partial x_i} E_j(0) | \psi_1 \rangle = -\frac{e}{2} \langle \psi_2 | T Q_2 | \psi_1 \rangle \sum_{p=-2}^{2} T E_{2p} \begin{pmatrix} J_1 & 2 & J_2 \\ m_1 & p & m_2 \end{pmatrix}(2.12)\]
where the reduced matrix element only depends on the spatial coordinates and is constant for the $J' \rightarrow J$ transition under consideration. $TQ_2$ and $TE_2$ are spherical tensors of rank 2 relating to the spatial and E-field parts of the matrix element respectively. They are defined by:

$$T_{2,0} = \frac{2}{3} \alpha_{zz} - \frac{1}{3} \alpha_{xx} - \frac{1}{3} \alpha_{yy}$$

$$T_{2,\pm 1} = \pm \sqrt{\frac{1}{6}} \left[ (\alpha_{xx} + \alpha_{xz}) \pm i(\alpha_{zy} + \alpha_{yz}) \right]$$

$$T_{2,\pm 2} = \sqrt{\frac{1}{6}} \left[ (\alpha_{xx} - \alpha_{yy}) \pm i(\alpha_{xy} + \alpha_{yx}) \right]$$

with

$$\alpha_{ij} = r_i r_j - \frac{1}{3} \delta_{ij} r^2$$

$$\alpha_{ij} = \frac{\partial}{\partial x_i} E_j(0)$$

The five terms of the sum in (2.12) represent the five possible $\Delta M_J$ transitions. Their relative transition rates are found by multiplying the $3 - j$ symbol in (2.12) for a given component with the associated $TE_{2,p}$ term and taking the modulus squared. The value of the tensor $TE_{2,p}$ depends on the direction of the quantization axis and the polarization state of the probe laser. A calculated spectrum for the the typical conditions used with the NRC trap is shown in Figure 2.2. The transition strengths were evaluated for a probe laser with linear
polarization state that is orthogonal to the plane defined by the quantization axis and the laser \( \mathbf{k} \) vector. The angle between the quantization axis and the laser \( \mathbf{k} \) vector is \( \approx 10^\circ \).

### 2.5 Hyperfine Structure

The nucleus of an atom has an associated spin \( \mathbf{I} \) that is the composite of the proton and neutron spin contributions from the nucleons that make up the nucleus. While \( \mathbf{I} = 0 \) for \(^{88}\text{Sr}^+\), this is not the case for other atoms such as \(^{85}\text{Rb}\) which was also studied in this work. A non zero nuclear spin has an associated dipole moment \( \mu_I \) which interacts with the magnetic field of the orbiting electrons \( \mathbf{B}_{\text{el}} \) as observed at the nucleus. This magnetic
moment is proportional to the nuclear spin $I$. For $I, J < 1$ the magnetic dipole term is the only significant contribution. The magnetic dipole term of the perturbing Hamiltonian for this interaction is [9]:

$$\mathcal{H} = - \mu_I \cdot B_{el}$$

$$= A I \cdot J$$

(2.19)

(2.20)

where, $J$ is the total orbital angular momentum and the magnetic dipole constant $A$ is an experimentally measured parameter. This interaction results in a splitting of the fine structure levels into hyperfine levels that are $(2F + 1)$-fold degenerate. The total angular momentum of the atom $F$ is the combination of the contributions from the total angular momentum of the electrons $J$ and the nuclear spin $I$. The allowed values for $F$ are:

$$F = J + I, J + I - 1, \ldots, |J - I|.$$  

(2.21)

The energy shift of a level $J$ resulting from the magnetic dipole interaction term in hyperfine structure is given by the expectation value of the interaction from (2.20). The resulting energy shift is:

$$\Delta E_{hfs} = \frac{1}{2} h AK$$

(2.22)

where $K = F(F+1) - I(I+1) - J(J+1)$ and $A$ is the magnetic dipole constant expressed in Hz.
3 Background Elements Concerning the Trapped and Laser Cooled $^{88}\text{Sr}^+$ Optical Frequency Standard

3.1 Fundamental Concepts of Ion Confinement and Laser Cooling

3.1.1 RF Paul Trap

To trap an ion, we wish to confine it within a sufficiently deep potential well that has a potential minimum near the center of a trap. The well depth should be such that it is much larger than the mean kinetic energy of the ion. Since an ion is a charged particle, it will experience a force in an electric field given by $F = q(E + v \times B)$. Unfortunately, the ion cannot be simultaneously confined in all three dimensions by the electrical potential produced by an arrangement of static electric charges. Such confinement is prohibited by Gauss’s Law [14] which requires the divergence of the electric field in a charge free region of space to be zero.

The solution to this is to create a time varying electric field whose time averaged “pseudo potential” is able to provide three dimensional confinement. An example of a trap that achieves this is the RF Paul Trap shown in Figure 3.1. This trap consists of a ring electrode and two endcap electrodes. A DC bias voltage $U_0$ as well as a time varying potential $V_0\cos(\Omega t)$ is applied between the ring and the two end caps as shown. The resulting instantaneous potential near the center of the trap has the form:
\[ \phi(r, z, t) = A(t)(r^2 - 2z^2) \]  \hspace{1cm} (3.1)

with,
\[ A(t) = \frac{U_0 + V_0 \cos(\Omega t)}{r_0^2 + 2z_0^2}. \]  \hspace{1cm} (3.2)

The stability of an ion's motion in this type of trap depends on its charge to mass ratio, the trapping potentials \( U_0 \) and \( V_0 \), the physical dimensions of the trap given by \( r_0 \) and \( z_0 \) (shown) and the applied oscillation frequency \( f = \frac{\Omega}{2\pi} \). For specific combinations of these parameters [15], the ion's motion in the trap will be stable and it will be confined in the effective pseudo potential of a three dimensional harmonic well. The strength of the trapping potential is not necessarily the same in all directions and can be resolved along the orthogonal directions of the trap, specifically an axial potential along \( z_0 \) and a radial potential along \( r_0 \). In practice, a real trap does not have perfect rotational symmetry and the radial confinement can be resolved into two orthogonal components described by an \( x \) and \( y \) axis both of which are orthogonal to the axial \( (z_0) \) direction. The confinement of the ion in this trap is analogous to that of a mass supported by three orthogonal springs, each with a different spring constant. Kinetic energy imparted to such a mass will result in motion at secular frequencies defined
by the three spring constants. For the ion, kinetic energy results in secular motion within the trap which in each orthogonal trap direction appears at a frequency determined by the depth of the trapping potential in that direction. The secular frequencies for the case of $U_0 = 0$ are [16]:

$$\omega_z = \frac{q\Omega}{2\sqrt{2}}$$  \hspace{1cm} (3.3)

$$\omega_r = \frac{q\Omega}{4\sqrt{2}}$$  \hspace{1cm} (3.4)

where,

$$q = \frac{8eV_0}{M\Omega^2(r_0^2 + 2z_0^2)}.$$  \hspace{1cm} (3.5)

In the above equations, $M$ is the ion mass and $e$ is the electron charge. The case of having no potential between the ring and the endcap electrodes ($U_0 = 0$) is typical for the NRC trap. This secular motion is used in Section 5.4 to determine the ion’s kinetic temperature. The ion can be displaced from the center of the trap by an external electric field such as a patch potential on one of the traps electrodes [17]. The effect of such a potential is to push the ion away from the center of the trap which causes it to undergo “driven motion” or micromotion as it interacts with the trap’s time varying electrical potential. This driven motion can cause heating of the ion as well as result in first order Doppler Broadening and second order Doppler shifts in the clock transition [18].

The parameters used for the NRC trap are summarized in Table 3.1. These result in a $^{88}\text{Sr}^+$ trap with a pseudopotential well depth of 5.5eV in the radial direction and 10.8eV in the axial direction. The resulting axial and radial secular frequencies are 1950kHz and 975kHz respectively.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0 = \frac{r_0}{\sqrt{2}}$</td>
<td>0.47mm</td>
</tr>
<tr>
<td>$\frac{\Omega}{2\pi}$</td>
<td>12MHz</td>
</tr>
<tr>
<td>$V_0$</td>
<td>265V</td>
</tr>
<tr>
<td>$U_0$</td>
<td>0V</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of parameters used in the NRC Paul trap.

3.1.2 Approach to loading

As described in the previous section, the ion trap produces a pseudopotential of several eV depth which can be used to confine the ion once inside the trap. Unfortunately the pseudopotential also represents a barrier resisting the passage of charged particles into the trap. Because of this, the successful loading of Sr ions into the trap requires that the ionization of the Sr atom take place near the center of the trap.

Neutral Sr atoms are produced by an oven located approximately 5mm from the center of the trap. The oven contains a powdered mixture of SrAl$_4$ and Ni. When heated, the powder melts and a reaction between the SrAl$_4$ and Ni produces neutral Sr as well as NiAl. A beam of Sr is emitted through a hole in the oven in the direction of the trap. At the same time the presence of liquid Sr on the surface of the oven reduces the workfunction of the metal allowing for thermionic emission of electrons from the oven surface. These electrons are accelerated toward the trap by a 90V potential that is applied between the oven and the trap electrodes. Periodically one of these electrons collides with a neutral Sr atom near the center of the trap causing the ionization of the atom. Once ionized, it is held in the harmonic potential of the trap and laser cooled. The ion fluorescence is detected at which point the oven is disconnected.
and the loading process stops. A detailed description of the type of oven used in the NRC $^{88}$Sr$^+$ trap is given in [19]

### 3.1.3 Doppler Laser Cooling

The typical temperature of an ion once captured and confined in the harmonic potential of an RF Paul trap is on the order of several thousand Kelvin [20]. The motion associated with this high temperature would make ultrahigh resolution spectroscopy of the $^{88}$Sr$^+$ ion’s clock transition impossible. To lower the ion’s temperature, it is cooled by the damping force [21], resulting from the repeated absorption and reemission of photons by the ion’s strongly allowed S-P “cooling transition”. The cooling radiation photons, with wave vector $k$, each carry momentum $p = \hbar k$. The optical frequency $\omega$ of the cooling radiation is detuned to lower frequency from the cooling transition’s line center $\omega_0$ by $\Delta \omega = \omega_0 - \omega$. In the reference frame of a moving ion, this cooling radiation will appear Doppler shifted closer to $\omega_0$, when the ion has a component of motion directed toward the light source, causing an increase in the absorption rate. Similarly, the absorption rate decreases when the ion is moving away from the cooling laser source. Each time a photon is absorbed it transfers its momentum to the ion which, for the detuning described above, will oppose the ion’s motion. For an ion held in a harmonic potential, it is sufficient to cool in one direction provided that the cooling laser has a projection onto each of the canonical directions defined by the trapping potential [22]. The isotropic emission of a photons during the ion’s spontaneous decay exerts no net force over the course of many absorption/reemission events. While the mean change in velocity resulting from reemission events is zero the mean square velocity is not. The effect of reemission can be thought of as a random walk in momentum space and imposes a limit on the lowest temperature that can be reached through Doppler laser cooling. For the case of Doppler laser cooling using a laser acting on a transition with linewidth $\frac{\gamma}{2\pi}$ the minimum
achievable temperature is [20]:

\[ T_{\text{min}} = \frac{\hbar \gamma}{4\pi k_B} \]  

(3.6)

For the \(5s^2S_{1/2} \rightarrow 5p^2P_{1/2}\) cooling transition in \(^{88}\text{Sr}^+\), the minimum kinetic temperature is approximately 0.5 mK. Results of temperature measurements of the \(^{88}\text{Sr}^+\) ion in the NRC trap are presented in Section 5.4. Through Doppler laser cooling it is possible to confine an ion in a region of space with diameter \(d\) such that:

\[ d < \frac{\lambda}{\pi}. \]  

(3.7)

Where \(\lambda\) is the wavelength of a laser source that is used to excite an electronic transition in the ion. If the so called Lamb-Dicke criterion 3.7 is satisfied, the observed transition frequency will not exhibit a first order Doppler Shift [23].

### 3.2 Frequency Stabilized Probe Laser System

The linewidth observed while probing the \(^{88}\text{Sr}^+\ 5s^2S_{1/2} \rightarrow 4d^2D_{5/2}\) clock transition is the convolution of the probe laser line width (in the reference frame of the ion) and the natural linewidth of the transition. To date, no group has been able to resolve the \(^{88}\text{Sr}^+\) clock transition down to the level of its natural linewidth. The probe laser system is one of the major elements of the \(^{88}\text{Sr}^+\) optical frequency standard. The linewidth and stability of the probe source is often the limiting factor in the performance of the standard. The current probe laser system at the NRC [24], consists of a commercial extended cavity (Littman-configuration) diode laser. It is frequency stabilized with a cascade of two Fabry-Perot (FP) resonators. The first is used for active stabilization of the diode laser, and also acts as an optical filter,
blocking optical noise outside the resonator 100kHz (FWHM) linewidth. The frequency of the filtered radiation emerging from the first resonator can be adjusted by changing the cavity spacing with a PZT actuator. The filtered radiation is then locked onto a second ultra narrow 3.7kHz (FWHM) resonator that consists of mirrors optically contacted onto an Ultra Low Expansion coefficient glass (ULE) spacer. The tremendous stability demonstrated by the probe laser system is a direct consequence of the mechanical stability of the ULE spacer. In a recent series of measurements [25] the absolute drift rate of the probe laser while locked to the ULE cavity was measured to be better than 10 mHz /s at an operating frequency of 445 THz. The result of scanning the laser frequency across the $^{88}$Sr$^+$ clock transitions is shown in Figure 5.7. A fit to this data indicated a 5.1Hz linewidth that combines contributions from the laser’s linewidth as well as possible broadening effects resulting from the transport of the beam to the ion as well as mechanical motion of the ion trap itself.

### 3.3 Optical Frequency Comb Fundamentals

The second is the duration of $9192631770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom [6].

Any measurement of the absolute frequency of a new standard needs to be in terms of the “second”, requiring a traceable comparison of the new frequency to that of the microwave transition defining the SI second. The frequency of visible radiation is approximately 50 000 times higher than that of a Cs clock making direct counting of optical frequencies using electronic means impossible [26]. Until recently, direct measurement of the absolute frequencies of optical transitions required the construction of elaborate optical frequency chains [27]. The NRC Optical Frequency Standards group was one of the few groups in the world to be successful in constructing a chain linking microwave and visible optical frequencies. In 1998 they reported a direct measurement of the absolute frequency of the $^{88}$Sr$^+$ clock transition
The complexity of optical frequency measurements changed in 1999 with the introduction of the optical frequency combs as a tool for making direct measurements of optical frequencies in terms of Cs microwave frequency standards [27]. A mode-locked laser produces a train of ultrafast (∼30fs) pulses at its repetition rate of $f_{\text{rep}} = 1/T_{\text{REP}}$. The Fourier transform of this train of delta function like pulses in the time domain is a series of evenly spaced comb elements in the frequency domain (Figure 3.2). Each comb element has a frequency $f_c = mf_{\text{REP}} + f_0$. The superposition of hundreds of thousands of comb elements all constructively interfere to produce each ultrafast pulse in the time domain. There is in general an offset between the phase of the carrier and that of the pulse envelope $\phi_{\text{CEO}}$, as indicated in Figure 3.3. This carrier envelope phase can change from pulse to pulse. The slippage of $\phi_{\text{CEO}}$ causes a frequency offset $f_0$ between the comb of frequency elements produced by a real mode-locked laser and a comb of elements starting from zero Hz by the offset beat frequency $f_0$.

In 2000, Jones et al. [4] showed how the carrier envelope phase could be detected by observing the beat frequency $f_o$ between the blue side of the comb and the frequency doubled red side of the comb. The absolute frequency of each comb element can be determined by stabilizing both $f_{\text{rep}}$ and $f_0$ and counting their respective frequencies with counters that are referenced to the SI second. The absolute frequency of an unknown laser can then be determined by
Figure 3.3: Schematic representation of the train of pulses emitted by an ultrafast laser. The pulses are separated in time by $1/f_{rep}$ and modulate a carrier of frequency $f_c$. The pulse to pulse carrier envelope phase is indicated as $\phi_{CEO}$ counting the heterodyne beat frequency between the unknown laser and the stabilized optical frequency comb. This measurement requires prior knowledge of the laser’s frequency to within less than the comb spacing which is usually several hundred MHz. This level of accuracy is easily achieved with commercial wavelength meters.

Using optical frequency combs it is possible, with averaging, to measure optical frequencies to the same accuracy as the time standard used as a microwave frequency reference. The development of optical frequency combs is a key enabling technology for the practical development and use of time standards based on optical transitions such as the $^{88}\text{Sr}^+$ based standard under development at the NRC. Because of the enormous impact of the frequency comb technique part of the 2005 Nobel Prize in Physics was shared by John Hall and Theodor Hänsch for its development. Chapter 6 will describe absolute measurements of the optical frequencies in Rb near 422-nm that were made using an optical frequency comb.
3.4 Measurement of Highly Forbidden Transitions in an $^{88}\text{Sr}^+$ Ion Trap

In Chapter 2 some of the basic quantum mechanical features of the $^{88}\text{Sr}^+$ ion were described. The chapter indicated that the ion can be laser cooled with radiation at 422nm exciting its $5s^2S_{1/2} - 5p^2P_{1/2}$ transition in conjunction with radiation at 1092nm which is required to pump the ion out of its metastable $^2D_{3/2}$ level. Once cooled, a ‘probe laser’ source at 674nm is used to attempt to excite the ion’s long lived ($\tau = 400\text{ms}$) ‘clock transition’. After probing, the cooling lasers are turned back on to resume laser cooling and detect whether or not a clock transition took place. The following section will describe the sequence for cooling the ion, exciting the 445 THz transition and then detecting when a transition has taken place. It will then describe the procedure for determining the field free line center of the $^{88}\text{Sr}^+$ $5s^2S_{1/2} - 4d^2D_{5/2}$ clock transition manifold. A diagram showing the the major elements of the $^{88}\text{Sr}^+$ trap experiment is given in Figure 3.4

3.4.1 Cooling-Probing-Fluorescence Detection Cycle for Detection of Highly Forbidden Transitions

The long natural lifetime of the clock transition makes direct detection of the 674nm photons scattered by the ion when the probe laser is tuned into resonance virtually impossible. At most, a few photons per second would be scattered, and with these photons emitted into $4\pi$ steradians, the detection rate for this already rare event with the system’s photomultiplier tube would be vanishingly small. In 1973 Dehmelt [1] suggested a solution to this problem by noting that while the ion is placed in a metastable level, transitions out of its ground state cannot occur. When the $^{88}\text{Sr}^+$ ion is ‘shelved’ in its $^2D_{5/2}$ level after having undergone
a clock transition, it cannot absorb photons from a cooling laser that is tuned to excite S-P transitions. The cooling transition is dipole allowed and normally scatters $10^6$ photons per second which allows for straightforward fluorescence detection using a photomultiplier based photon counting system. A short interruption in the ion fluorescence is easily detected and indicates that the ion has become shelved in its long lived state. This fluorescence detection technique allows for nearly 100% detection of quantum jumps.

The sequence for cooling, probing and then fluorescence detection of the electronic state of the ion is as follows. First the ion is cooled with $\sim 8\muW$ of radiation at 422nm, focused near the center of the trap, with a spot size of $\omega = 15\mu$m, exciting S-P transitions. A second repump laser (Koheras Adjustik Y10 PZT) produces $50\muW$ of radiation at 1092nm that is focused to

Figure 3.4: Diagram of major components of $^{88}\text{Sr}^+$ trap experiment.
\( \omega = 60\mu m \). This laser pumps population out of the \( ^2D_{3/2} \) level by exciting D-P transitions. These lasers interact with the ion for a period of approximately 25ms and are then both blocked in such a way that the 1092nm source irradiates the ion for a short time after the 422nm source is blocked. This ensures that the any population in the \( ^2D_{5/2} \) level is returned to the ground state. It is essential that both of these sources are blocked prior to probing the clock transition in order to avoid power broadening of the clock transition as well as a Stark shifts of the levels. Once the ion has been cooled, probe radiation at 674nm is focused onto it with a spot size of \( \omega = 30\mu m \) in an attempt to excite a clock transition. The typical probe powers used range from a few microwatts to hundreds of picowatts depending on the which line is scanned and the probe pulse duration that is used. The probe laser interrogates the ion for a predetermined period of time, which for interaction times less than \( \sim 200\text{ms} \) broadens the clock transition and a Fourier transform limited linewidth is observed [29]. Following the probe pulse, the cooling cycle resumes. As soon as cooling begins, the fluorescence rate from the ion is detected by imaging the ion fluorescence into a photomultiplier tube with f/2 optics. While cooling, a fluorescence rate of \( \sim 7000 \text{ counts/second} \) is typically observed relative to the 200 counts/second background. A plot of the fluorescence rate at the start of each cooling cycle is shown in Figure 5.3 (p. 78). Quantum jumps are clearly visible in this plot where the fluorescence is interrupted for several cooling/probing cycles as the result of the \(^{88}\text{Sr}^+\) having undergone a clock transition.

This cooling/probing/fluorescence detection cycle repeats for a set period of time and the rate of observed quantum jumps is recorded with a computer. The computer can be programmed to scan one or more transition lines by changing the probe laser frequency using the AOM shown in Figure 3.4 with predetermined frequency steps and then measuring the excitation rate for a set period of time at that frequency step. An example of the lineshape for a clock
transition scanned using this technique is shown in Figure 5.7 (p. 85).

### 3.4.2 Determination of Field Free Line Center

As $^{88}\text{Sr}^+$ is an even isotope ($I = 0$) and $M_j \neq 0$, all of the S-D transitions are shifted as a result of the presence of the magnetic field inside the trap. In order to estimate the field free line center of the S-D transition, the line centers of symmetric pairs of Zeeman components are measured (see for example Figure 2.2). The average of the transition frequencies of a given pair of Zeeman components is equal to the S-D transition’s field free line center to first order in the Zeeman perturbation. Higher order contributions from the (10µ T) magnetic fields in the trap are on the mHz level or smaller [18].

The absolute frequency of the probe laser prior to the AOM shown in Figure 3.4 is determined by directing some of the probe laser radiation to an optical frequency comb. The comb is used to measure the probe optical frequency using the approach described in Section 3.3. Both the determination of the clock transition field free line center and the measurement with the frequency comb require several minutes to complete. The accuracy of this experiment relies on the stability of the probe laser over this extended period. As described in Section 3.2, the probe laser is locked to a ULE cavity which allows it to have an extremely small and well characterized drift rate of on the order of 10 mHz/sec. After passing through the AOM, the probe laser’s optical frequency can be adjusted by changing the RF frequency driving the AOM. This AOM frequency is controlled using a computer and a synthesizer that is referenced to the Institute’s realization of the second. By knowing the absolute frequency of the probe before the AOM, as well as the AOM’s drive frequency, the absolute frequency of the probe radiation directed at the ion is known.
The AOM frequency that tunes the probe to the clock transition’s field free line center is determined by successively probing each Zeeman component at two points on either side of its line center. Determining the AOM frequencies that would balance the jump rates on either side of line center enables the computer to infer where the line center is. By determining the line center frequencies of a symmetric pair of Zeeman components and averaging them, the AOM frequency that would correspond to the clock transition field free line center is determined. This AOM frequency is then added (or subtracted depending on how the AOM was arranged) from the measured absolute optical frequency of the probe laser before the AOM. The result of this is the determined frequency of the $^{88}$Sr$^+$ clock transition.

Absolute measurements of the clock transition’s field free line center were not carried out as part of this thesis. The most recently reported measurement of this transition frequency by the NRC $^{88}$Sr$^+$ group [30] was published in (2005) as $f = (444\,779\,044\,095\,484 \pm 15)$ Hz.
4 Design of New Cooling Laser at 422nm

The recent availability of GaN laser diodes which operate in the violet region of the spectrum has enabled the development of a new laser cooling source where the radiation is produced directly by the laser diode at 422nm as opposed to relying on the frequency doubling of radiation from a near infrared laser diode. The advantages of this approach when compared to systems that use frequency doubled radiation include a reduction in technical complexity as well as the ability to obtain significantly more optical power at the 422nm cooling transition wavelength. This increased power is needed for future ion trap experiments at the NRC involving the simultaneous operation of multiple ion traps each employing a three beam cooling geometry (compared to the single beam cooling geometry employed in the current generation trap). The increased power available from these lasers also allows for saturation spectroscopy of a $^{85}\text{Rb}$ reference transition located near the $^{88}\text{Sr}^+$ cooling transition frequency. Saturating the $^{85}\text{Rb}$ atomic reference allows for a frequency servo lock to the atom’s Doppler free line center yielding much greater reproducibility and control of the optical frequency of the cooling radiation compared to previous results obtained with frequency doubled radiation locked onto the broad $^{85}\text{Rb}$ Doppler profile.

4.1 Review of Old Frequency Doubled 844 nm Laser System

The preliminary experiments at the NRC with the $^{88}\text{Sr}^+$ ion were performed with cooling radiation from a dye laser (Coherent-699) [31]. This laser radiation had a $\sim$ 1MHz linewidth,
however as it was not locked to an atomic reference, it demonstrated poor long term stability. The significant effort required to keep a dye laser operating for extended periods of time motivated the development of a cooling source based on frequency doubling a near infrared laser diode.

The frequency doubled 422nm laser source at the NRC was described in [32], which also described the use of the use of the $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 2, 3)$ transition in neutral $^{85}\text{Rb}$ to stabilize the laser’s frequency near the cooling transition in $^{88}\text{Sr}^+$. The frequency doubled source consists of an external cavity diode laser (New Focus Model 6226) which produces 14mW of radiation at 843nm. This source is frequency doubled with a $\text{KNbO}_3$ crystal located inside a triangular resonator cavity. The cavity gives a calculated enhancement factor of 20, greatly increasing the production of 422nm radiation in the nonlinear crystal. When first employed in 1998, the system was able to produce 130$\mu$W of power at 422nm. At present the nonlinear crystal is believed to have become partially depolled and the resulting reduction in $\chi^{(2)}$ susceptibility has reduced the system’s output power to around 40$\mu$W.

In order to stabilize the frequency of this laser near the cooling transition in $^{88}\text{Sr}^+$, it was locked onto the $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 2, 3)$ transition in $^{85}\text{Rb}$ vapor. The amount of 422nm power produced by the doubled system was sufficient for cooling the ion however it was not sufficient to saturate the atomic $^{85}\text{Rb}$ reference. Instead, a small fraction of the 422nm beam ($1\mu$W) was used to lock onto the Doppler broadened $^{85}\text{Rb}$ absorption profile. This lock was achieved by frequency shifting part of the 1$\mu$W beam in an AOM. The two physically separate, shifted and unshifted beams passed through a heated $^{85}\text{Rb}$ cell and their transmitted intensities were detected with separate photodetectors. The laser’s frequency was tuned so that the shifted and unshifted beams interacted with Rb atoms on opposite sides of
the Doppler broadened Rb absorption profile. The laser’s frequency was then adjusted by a feedback loop which maintained a constant transmitted power difference between the shifted and unshifted beams. This maintained the unshifted beam and thus the 422-nm beam that went to the ion trap experiment at a fixed detuning from the linecenter of the Rb reference transition. By setting this detuning close to the $\sim 440\text{MHz}$ difference between the frequency of the $^{85}\text{Rb}$ reference and that of the $^{88}\text{Sr}^+$ cooling transition, the 422nm source was held at a chosen detuning from the $^{88}\text{Sr}^+$ cooling transition.

The frequency doubled system was used successfully with the NRC ion trap experiment for $\sim 8$ years, however it did have several limitations. The technical complexity of this system caused it to require significant operator attention to run. Also, as the limited optical power available from this system was insufficient to saturate the $^{85}\text{Rb}$ reference transition, it was only possible to lock the laser onto the transition’s broader Doppler profile. This type of lock is much more sensitive to small variations in parameters such as the temperature/pressure of the Rb vapor yielding greatly reduced frequency stability when compared to a system that is able to be locked onto the Doppler-free saturated absorption profile of a transition. The method of locking onto the side of the Doppler profile also led to uncertainty as to the exact detuning of the cooling laser from the reference transition’s line center. This ambiguity as to the absolute frequency of the cooling laser greatly complicated the procedure for loading the ion trap as it could not be guaranteed that the cooling source had the correct red detuning from the $^{88}\text{Sr}^+$ cooling transition’s line center that is needed for cooling of a trapped $^{88}\text{Sr}^+$ ion. Finally, while the frequency doubled system produced sufficient 422nm power to cool the ion in the existing ion trap, a new trap is currently under development which calls for a three beam cooling geometry. A cooling laser source with greater optical power is desirable for this new cooling configuration.
4.2 Description of the New Diode Laser at 422nm

The new cooling laser system is based on a commercial External Cavity Diode Laser (ECDL) (Toptica DL100) which employs a GaN Laser Diode (LD) and a diffraction grating mounted in the Littrow configuration [33], which provides frequency selective feedback into the laser diode.

A diode laser’s wavelength is determined primarily by the semiconductor’s bandgap and by the junction’s temperature and current density. In practice, the bandgap is determined by the semiconductor material and by the manufacturing process. Small (∼20nm) adjustments to the laser’s wavelength can be made by adjusting the diode’s temperature and current. Changes to temperature affects both the optical path length of the laser cavity as well as the wavelength dependence of the laser’s gain curve. Adjusting the current density affects both the diode’s temperature as well as the carrier density, and as the diode’s refractive index depends both on its temperature and its carrier density, a change in current will cause a change in operating wavelength. For time scales longer than 1µs, the temperature effect on current tuning dominates over the carrier effect and changes to carrier density can be ignored. A detailed description of temperature and current tuning is given in [34].

A typical single mode laser diode with a cavity defined by the two end facets of the diode will exhibit a linewidth of tens to hundreds of MHz [34]. This can be reduced by placing the diode in an external cavity which increases the photon lifetime in the cavity thus reducing...
the emission linewidth via reduction of Schallow-Townes spontaneous emission noise [35]. A frequency selective element in the external cavity can be used to increase the tuning range of the diode by forcing it to operate at wavelengths away from the peak of its gain curve. The frequency selective element in the DL100 is a diffraction grating mounted in the Littrow configuration.

In the Littrow configuration, the emission from a laser diode is collimated and then directed onto a diffraction grating. The angle of the diffraction grating is chosen so that the grating’s first order diffracted beam is directed back into the LD forming an optical resonator and providing optical feedback to the diode. The zero order diffracted beam (reflection) from the grating is used to couple radiation out of the cavity. When placed in a cavity, the grating will pull the laser’s wavelength toward the grating’s minimum loss wavelength, $\lambda_r(\theta)$ which is a function of the grating line spacing as well as the angle of the grating with respect to the incident beam. As with other types of lasers, an ECDL will tend to lase on a longitudinal mode where an integer number of wavelengths are able to fit between the cavity mirrors. Thus the wavelength of the resonant cavity mode is given by $\lambda_q(\theta) = \frac{2L}{q}$, where $q$ is the mode number and $L$ is the effective cavity length. The frequency selectivity of a Littrow ECDL depends on the two factors of the grating’s angle and the optical path length of the resonator formed between the grating and the rear facet of the LD. The laser’s wavelength can be tuned by translating and or rotating the grating. However if the rate of change of $\lambda_q(\theta)$ and $\lambda_r(\theta)$ do not match, the laser will have a tendency to modehop after tuning over a small range of wavelengths. A solution to this is presented in [36] where the pivot point for the grating’s motion is chosen such that both the grating angle and the optical path length change at the same time and both $\lambda_r(\theta)$ and $\lambda_q(\theta)$ change at the same rate. This situation is depicted in Figure 4.2.
In the DL100 laser, the diode temperature is stabilized and was used to bring the laser’s wavelength close to the that of the $^{88}\text{Sr}^+$ cooling transition. Fine frequency adjustments were made by changing the position of the laser’s diffraction grating with a piezoelectric transducer (PZT) which changes the operating wavelength as previously described. As will be described in the following subsections, the laser’s wavelength is actively stabilized by making low frequency ($\sim 3\text{kHz}$) adjustments to the PZT voltage and high frequency adjustments (with a bandwidth of up to $\sim 590\text{kHz}$) to the diode current.

The free-running linewidth of the DL100 at 422nm was found to be $\sim 10\text{MHz}$, which was considerably worse than the $< 1\text{MHz}$ linewidths demonstrated by Littrow ECDL’s operating in the NIR [37] as well as the $\sim 1\text{MHz}$ typical linewidth quoted by the manufacturer [38]. The performance of this system was consistent with the experiences reported by other authors with Toptica DL100 lasers using GaN-based LD’s at 410nm [39–41]. Authors employing AR coated diodes reported greatly improved short term (50ms) linewidths of 0.8 MHz [42]. Unfortunately, anti-reflection coated diodes at 422nm were not available. The only known publication where an author reports using a GaN LD at 422nm [43] involved the use of resonant optical feedback from a confocal cavity, which was able to reduce the laser linewidth to less than 880kHz. This technique was not used for the new cooling laser system as its requirement of maintaining fraction of a wavelength alignment between the laser and cavity was considered to be too demanding given the requirement of the cooling laser to stay locked.
for periods of several hours.

4.3 Laser Line Narrowing

The DL100’s initial $\sim10$MHz linewidth was considered to be inadequate for efficient cooling of the $^{88}\text{Sr}^+$ ion due to the similar widths of the laser’s linewidth and that of the ion’s cooling transition. The laser’s linewidth would also limit the reproducibility of a Rb frequency reference which was also developed as part of this work. For these reasons, the ECDL was prestabilized onto a Fabry-Perot (FP) cavity resonance.

4.3.1 Laser Frequency Noise

A laser’s frequency is broadened by both deterministic as well as random noise sources. Deterministic sources include effects such as electrical pickup by the laser diode from AC sources such as the mains, mechanical vibrations in the lab as well as pressure fluctuations due to acoustic noise. Random noise sources tend to be more fundamental and include thermal noise in the laser electronics and optical components, as well as Schawlow-Townes noise.

To understand the effect of these noise sources, it must first be understood that the laser’s optical frequency $\nu$ is not constant but varies about its temporal mean $\bar{\nu}$ with instantaneous frequency $\nu(t)$ [23]. The instantaneous frequency fluctuation is given by $\Delta\nu(t) = |\bar{\nu} - \nu(t)| << \bar{\nu}$. Noise sources and the electronic feedback methods used to minimize their effects are best understood by looking at the Power Spectral Density (PSD) of the instantaneous frequency fluctuations. The power spectral density $S_{\nu}(f)$ is calculated using the Wiener-Khinchine theorem [44], which states that the autocorrelation of a function in the time domain is proportional to Fourier Transform of that function’s PSD. For instantaneous fluctuations in laser frequency the autocorrelation is given by:
\[ R_\nu(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \Delta \nu(t + \tau) \Delta \nu(t) dt \]  

(4.1)

and the resulting PSD is calculated as:

\[ S_\nu(f) = \int_{-\infty}^{\infty} R_\nu(\tau) e^{-i2\pi f \tau} d\tau. \]  

(4.2)

This analysis shows that the instantaneous fluctuations in the laser optical frequency can be represented as the superposition of a continuous distribution of Fourier terms each acting at a frequency \( f \) with noise power \( S_\nu(f) \). Expressing \( \Delta \nu(t) \) in terms of its Fourier components simplifies noise analysis and servo design as different noise sources dominate at different Fourier frequencies and the ability of an electronic feedback circuit to reduce the laser’s noise depends on the range of frequencies or bandwidth over which the servo operates.

In order to detect fluctuations in a laser’s frequency, the fluctuations must be converted from changes in the laser’s optical field to a proportional electrical signal. One approach to this is to combine radiation from the laser with that of a second laser of similar optical frequency and then observe the heterodyne beat between the two sources on a photodetector [35]. Another approach is to direct the laser into a frequency discriminator such as a Fabry-Perot cavity [33]. When the laser is tuned the half power point of the Fabry-Perot cavity’s transmission fringe, a small change in laser frequency \( \Delta \nu(t) \) will cause a proportional change in the intensity of the light transmitted through the cavity. This transmitted light can be detected with a photodetector which will convert the varying light intensity into an electrical signal that is proportional to \( \Delta \nu(t) \) [45]. Both of these approaches were used in this work and will
be discussed further in the following sections.

The frequency fluctuations of the DL100 were measured using a Fabry-Perot cavity in the arrangement described in Section 4.3.3.2. The resulting electrical signal was analyzed at frequencies between 0 Hz and 100 kHz with a digital Fast-Fourier Transform based spectrum analyzer (Stanford Research SR760) and at higher frequencies between 25 kHz and 1 MHz using a Radio Frequency (RF) spectrum analyzer (Advantest R3261B).

![Figure 4.3: Power Spectral Density of DL100 observed by tuning the laser onto the side of a Fabry-Perot transmission fringe and examining the intensity fluctuations of the transmitted light with a spectrum analyzer. The noise contributions are the strongest in audio frequency band up to a few kHz (left). At higher frequencies (right) the noise power decreases with a 1/f dependence up to a corner frequency of 270 kHz. For frequencies above 270 kHz the noise is primarily white with a constant frequency dependence. Note that noise powers shown are relative and do not equal the absolute power spectral density of the lasers frequency noise](image-url)

Figure 4.3 shows the measured PSD of the DL100. The vertical axis of these plots indicate the relative noise power of fluctuations of $\Delta \nu(t)$ with respect to Fourier frequency. While the absolute noise power is not given, these plots are useful for seeing the distribution of noise sources affecting the laser. As seen in Figure 4.3 a) there is a large contribution at low fourier frequencies up to a few kHz. This includes most technical noise sources such as
mechanical vibrations and acoustic disturbances. Electrical disturbances also contribute at low frequencies but continue to play a role at frequencies in the hundreds of kHz. Technical noise typically has an $f^{-\alpha}$ dependence as is seen in Figure 4.3 b). This system has a technical noise contribution that decreases with $\alpha = 0.97$ until a corner frequency is reached at $f_c = 270$ kHz. For frequencies higher than 270 kHz, the spectrum becomes nearly flat ($\alpha = 0.18$). This flat frequency response is characteristic of white frequency noise and for lasers, this contribution represents the quantum noise limit [46] imposed by Schawlow-Townes noise. Schawlow-Townes noise is caused by the lasers spontaneous emission noise randomly perturbing the phase of the carrier field.

4.3.2 Passive Stability

The laser linewidth is strongly perturbed by any mechanical vibration, air pressure change or temperature change that alters the optical path length of the laser’s external cavity. While it is sometimes possible to detect and cancel the effect of these perturbations with active stabilization of the laser’s optical frequency, (the topic of the next subsection) it is helpful to isolate the laser as much as possible from these influences. For the case of the cooling laser, this consisted of using the manufacturer supplied temperature controller to stabilize the diode’s temperature. To reduce its sensitivity to mechanical vibration, the ECDL was bolted to a large block of aluminum which along with the other optics for the cooling laser system was rigidly mounted to a four inch thick optical breadboard. The breadboard sits on an array of Viton® O-rings which have been shown to significantly dampen mechanical vibrations and the entire system is housed inside a plexiglass enclosure which reduces air currents and to some extent reduces the effect of acoustic disturbances.
4.3.3 Active Stabilization using a Fabry-Perot Reference Cavity

In Subsection: 4.3.1 the PSD of the DL100 was shown to have frequency components with a $f^{-1}$ dependence extending to Fourier frequencies in the hundreds of kilohertz. The overall laser linewidth is proportional to the integral of the PSD so to minimize the laser’s linewidth, it is desirable to reduce $S_\nu(f)$ over as wide a range of fourier frequencies as possible. So far, it has been shown that frequency fluctuations can be detected by using a Fabry-Perot cavity as a frequency discriminator. Furthermore, Section 4.2 stated that changes to the ECDL’s grating position using a PZT actuator as well as changes to the laser’s current will result in a change in its operating wavelength. This section will describe the use of an active feedback loop that first senses laser frequency deviations using a Fabry-Perot cavity based discriminator and then electronically adjusts the laser frequency to cancel these fluctuations.

4.3.3.1 Theory of Negative Feedback

The field of feedback and control is well developed with many good references in particular [47] as well as [48,49], also several authors have directly treated the application of feedback to laser stabilization [23,46,50]. The basic idea of negative feedback is to detect small changes in the output of a system from its desired operating point and to use a feedback loop to steer the system’s output to compensate for the changes. A conceptual model for this type of feedback loop is shown in Figure 4.4. The free running frequency of the laser is given as $\nu_i$. The desired operating frequency of the laser is the setpoint frequency $\nu_o$. The action of the servo is to steer the laser’s frequency to be as close as possible to the setpoint. The laser frequency when controlled by a feedback loop is distinguished from its free running frequency as $\nu_s$. 

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Deviations of the stabilized laser’s frequency from the setpoint defined as \( \delta \nu \equiv \nu_s - \nu_o \), are detected with a frequency discriminator which outputs an error signal \( \delta \nu D(f) \). The frequency dependence of the discriminator accounts for the frequency dependent gain and phase characteristics of the detection system. This error signal then passes through a servo loop filter \( G(f) \) which shapes the gain and phase characteristics of the feedback loop. The servo signal is then applied to an actuator \( K(f) \) which is able to effect the laser’s output frequency in response to the servo signal. The actuator has its own frequency dependent gain and phase characteristics which must be accounted for in the design of the servo loop filter’s response. The loop transfer function can be evaluated by conceptually breaking the feedback loop at any point and solving for the response of the loop to a test signal injected into one side of the break, as observed on the other side. For the case shown above, the loop transfer function is \( D(f)G(f)K(f) \). Since the goal of feedback is to reduce errors in the output frequency of the laser, the feedback signal applied by the actuator should move the laser’s frequency to counteract the disturbance (negative feedback). The resulting output of the stabilized laser is its free running frequency minus the feedback signal:
\[ \nu_s = \nu_i - \delta \nu D(f)G(f)K(f) \]  

Subtracting the setpoint frequency from both sides we get:

\[ \nu_s - \nu_o = \nu_i - \nu_o - \delta \nu D(f)G(f)K(f) \]  

The left hand side is the deviation of the stabilized laser from the reference cavity. Substituting \( \Delta \nu = \nu_i - \nu_o \) for the deviation of the free running laser from the reference cavity and rearranging the above equation in terms of \( \delta \nu \) we find:

\[ \delta \nu = \frac{\Delta \nu}{1 + D(f)G(f)K(f)} \]

This indicates that feedback can reduce errors by a factor of \( 1 + D(f)G(f)K(f) \) but is not capable of forcing them to zero. From this analysis, it is apparent that in order to minimize errors, a servo should be designed with the largest gain possible. The maximum usable gain is limited by time delays and the frequency dependent phase shifts that are present in any system. To see the effect of a phase shift, consider the case of a sinusoidal noise source \( S_{f,Laser} \) perturbing the laser carrier frequency. This would be detected by the discriminator and ideally the servo would apply a mirror image of the disturbance to the laser through the actuator canceling its effect. Small errors between the actual disturbance and the applied compensating signal from the actuator would also be detected and canceled by the feedback loop. The loop will work to minimize \( \delta \nu \) provided that the signal applied by the actuator has the opposite sign to that of the disturbance (negative feedback). Unfortunately the propagation time through the discriminator, servo and actuator adds a phase shift between the disturbance and the response of the actuator. If the loop gain is greater than unity when the phase shift is greater than 180°, the effect of feedback will be to amplify the disturbance;
in some cases the feedback loop will undergo self sustaining oscillations. To prevent this, the servo’s frequency dependent gain must be chosen so that the loop gain is less than one at the frequency $f_g$ where delays in the feedback loop cause a phase shift greater than $180^\circ$.

The ideal situation of having infinite gain for all frequencies below to $f_g$ and no gain for frequencies greater than $f_g$ is excluded by the “Kramers-Kronig” relations which describe the intrinsic coupling between the servo loop’s phase shift at frequency $f$ and the slope of the gain/attenuation function in the vicinity of $f$ [50]. In practice, if the gain of the loop transfer function rolls off at 6 dB /octave there will be a $90^\circ$ phase shift and a 12 dB / octave roll off will cause a $180^\circ$ shift. Therefore, having the loop gain decrease at a rate any faster than 12 dB / octave will cause the feedback loop to amplify noise over the range of frequencies where the loop gain is greater than unity. The maximum loop gain is dictated by the requirement that it must decrease at not more than 12 dB / octave and reach unity gain before $f_g$. The specific case of the servo designs used for this laser system will be discussed later in this section.

### 4.3.3.2 Use of a Fabry-Perot as a Discriminator

The practice of stabilizing a laser’s frequency by locking it onto the side of the transmission fringe of a stable Fabry-Perot (FP) cavity has been used by many authors [45,50], and has also been employed in commercial laser systems. A schematic representation of this technique is shown in Figure 4.5. The basic idea is that the laser is tuned to the half power point of the FP’s transmission fringe and the light passing through the FP is detected with a photodetector. Any change in the laser’s operating frequency (or the transmission frequency of the FP) will cause a change in the amount of light reaching the photodetector. Because
the laser is tuned to the half power point of the FP, the detected signal will either increase or
decrease depending on whether the laser’s optical frequency has increased or decreased. The
signal detected after the FP will be sensitive to any frequency modulation of the laser (FM)
as well as any change in the laser’s amplitude (AM). In order to derive an error signal which
only depends on frequency fluctuations, a second detector is used which detects changes in
the laser intensity. By balancing the signals from these two detectors and subtracting them,
an error signal is derived that is proportional to $\delta \nu$.

![Diagram of fringe locking technique]

**Figure 4.5:** Schematic representation of the side of fringe locking technique. A) Frequency deviations are detected by passing some of the laser light through a Fabry-Perot interferometer. A second detector senses amplitude fluctuations in the lasers intensity. B) Small changes in the laser’s frequency cause a proportional change in the light intensity transmitted through the Fabry-Perot

The Fabry-Perot used for this system consisted of two dielectric mirrors each with a focal
length of $f = \sim 30$ cm held 7.5 cm apart by an Invar® spacer. A Piezo Electric Actuator
(PZT) on one end of the spacer allows for adjustments of the cavity length. The mirrors were
obtained from an earlier experiment, so it was not possible to match their focal length to
the cavity spacing. Because the focal length are larger than the cavity spacing, modes other
than the fundamental TEM00 mode will allow transmission at other frequencies through the
cavity [51]. The discriminator signal found by sweeping the FP cavity length and observing the transmitted signal while subtracting the laser’s AM contribution is shown in Figure 4.6. The higher order modes are visible on the left side of the central TEM00 peaks. The free spectral range of this cavity was measured to be 2 GHz and the main peaks are 49 MHz (FWHM) wide, resulting in a cavity finesse of $F = 41$.

![Signal from the Fabry-Perot based frequency discriminator. The free spectral range is 2.0 GHz and the TEM00 fringe width is 49 MHz giving a cavity finesse of 41.](image)

**Figure 4.6:** Signal from the Fabry-Perot based frequency discriminator. The free spectral range is 2.0 GHz and the TEM00 fringe width is 49 MHz giving a cavity finesse of 41.

This cavity has been used to derive the signal that was used to stabilize the DL100. The performance of this cavity is limited by its relatively low finesse ($F = 41$) which limits the sensitivity of the error signal to changes in the laser’s frequency. A more significant limitation is noise added by the FP to the error signal due to temperature and pressure fluctuations in its ambient environment. Far better environmental isolation could be achieved by placing the FP in vacuum.
4.3.3.3 Frequency Control with Diffraction Grating “PZT Slow Servo”

Once frequency fluctuations are detected using the discriminator, the laser frequency is adjusted by a feedback loop in an attempt to cancel the fluctuations. For locking the DL100 laser onto the FP transmission fringe, two “Actuators” are employed to adjust the laser optical frequency. For low frequency noise components from 0Hz to $\sim 3\text{kHz}$, adjustments are made by changing the ECDL’s grating position with its PZT actuator. This “Slow Servo” is described in this subsection. A faster servo which operates by modulating the laser current is described in the next subsection.

As was described earlier, the ECLD’s frequency can be tuned by applying a voltage to its PZT actuator which changes the laser’s grating position. For DC voltages, the PZT causes a $534\text{MHz}/\text{V}$ change in the optical frequency of the laser. Using this actuator, the laser frequency can be changed by more than 10 GHz. To use this actuator as part of a feedback loop, a loop filter $G(f)$ was designed to maximize the servo gain while insuring that the loop gain drops to less than unity for frequencies where the loop’s phase shift is greater than 180°.

The transfer function for the DL100’s PZT actuator at a given frequency can be measured by applying a sinusoidal stimulus to the PZT and observing the amplitude and phase of the resulting signal from the discriminator. To measure the response across a range of frequencies, an audio spectrum analyzer (Hewlett Packard 3582A) was used which provided the stimulus to the PZT and analyzed the resulting signal from the discriminator. The transfer function of the DL100’s PZT actuator is represented in the Bode plot [49] in Figure 4.7.

This plot shows that the PZT based actuator reaches its first mechanical resonance at 3.2kHz. Resonances are observed in all mechanical systems that are not heavily dampened [52]. At resonance, the amplitude response rapidly increases as the phase shift passes through 180°.
In order to use this PZT as the actuator in a control loop, the loop gain must drop below unity prior to this resonance. The loop filter designed for this servo consists of a one pole integrator. Details of this type of filter are discussed in Appendix B. The integrator has nearly infinite gain at DC with a response that rolls off at 6db / octave, imparting a 90° phase lag to the loop transfer function. The overall phase shift of the loop transfer function is the sum of the phase shifts imparted by each element in the loop. The discriminator has a nearly flat magnitude response up to frequencies in excess of 1MHz and makes a marginal contribution to the phase shift at frequencies lower than 1MHz. For this feedback loop, the phase shift nearly equals the sum of the shifts from the actuator and the loop filter. Although the PZT actuator has its first resonance at 3.2 kHz, its phase shift reaches 90° at 2.0 kHz. If the action of the integrator were to continue to roll off at 6db/octave at this frequency, its 90° phase shift would combine with that of the actuator and cause instability in the servo loop if the loop gain were greater than unity. This would then limit the maximum loop gain as well as the range of frequencies over which the servo was able to operate. The performance
of the loop filter can be improved by adding a zero to the transfer function which flattens out its response at 1.9kHz. The effect of this is that the integrator’s phase shift is less than 90° and tending toward zero by the time the actuator’s phase shift reaches 90°.

![Amplitude and phase response of PZT based actuator with and without the calculated response of the loop filter.](image)

**Figure 4.8:** Amplitude (A) and phase (B) response of PZT based actuator with and without the calculated response of the loop filter. In each plot the actuator transfer function $K(f)$ is the top trace and the transfer function for the actuator convolved with the loop filter $K(f)G(f)$, is the lower trace. The Shaped frequency response reaches a 180° phase shift at 3.2kHz. The loop gain is set empirically in order to find the best trade off between maximizing the loop bandwidth and the minimizing the contribution from the noise peak resulting from the mechanical resonance at 3.2kHz.

By shaping the loop transfer function in this way, the servo’s closed loop gain is maximized and the servo is able to act to reduce noise over a range of frequencies nearly reaching the first mechanical resonance of the actuator. A Bode plot of the actuator response after convolution with the loop filter is shown in figure 4.8. The plot shows that the phase shift does not reach 180° until the first resonance is reached at 3.2kHz. The loop filter has caused the gain to roll off nearly 6db/octave faster than the unfiltered actuators response allowing much higher gain to be used at DC while still ensuring that that the loop gain drops below unity before the phase shift reaches 180°. This plot also shows that there is a significant increase in gain
\~13dB near the mechanical resonance. This peak will cause a ‘noise bump’ in the laser noise PSD if the loop gain is above unity at this point. Keeping this peak below unity gain would require the unity gain frequency to be set at around 650Hz which would not take advantage of the servo bandwidth between 650Hz and the and $f_g$ at 3.2 kHz. In practice, the loop gain is set empirically to minimize the total noise on the error signal. The result of this is that the noise bump does contribute noise by reaching above unity gain but its contribution to the total noise power is negated by the large increase in gain at low frequencies resulting from increasing the unity gain bandwidth beyond 650Hz. A plot showing the effect of this servo loop on the discriminator error signal is shown in 4.9. The noise bump from the mechanical resonance is visible at 3.2kHz in this plot. The circuit used to realize this servo is given in Appendix C.

![Graph showing effect of slow servo on error signal observed with FP discriminator](image)

**Figure 4.9:** Effect of slow servo on error signal observed with FP discriminator
4.3.4 Frequency Control with Current

The slow servo described above is adequate for holding the laser frequency at the lockpoint on the FP cavity and removing some low frequency noise. Unfortunately, this servo is not capable of reducing noise at Fourier frequencies greater than its first resonance at 3.2 kHz. In practice, because this servo’s response rolls off at 6dB / octave, it has little effect on noise contributions with Fourier frequencies above $\sim 200$Hz. To attack noise at higher Fourier frequencies, a servo was developed that takes advantage of the far greater servo bandwidth available through direct modulation of the laser diode’s current. The drawback of using current modulation is that it can cause significant amplitude fluctuations on the laser output compared to PZT based control described in the previous section. For this reason, the servo loops for the laser were designed so that the loop using the PZT based actuator contains an integrator giving it nearly infinite gain at DC, which allows it to completely dominate over the effect of current control at low frequencies. This ensures that the PZT controller is used to steer the frequency of the laser on long time scales making the large adjustments needed in order for the system to remain locked as the temperature of the room changes throughout the day. Current control is used to counteract what are usually very small high frequency excursions of the laser frequency from the lock point enabling a significant reduction in noise at higher Fourier frequencies. Because the current control loop mostly corrects for small frequency excursions, it adds minimal amplitude noise to the laser.

The laser diode is powered by a current source which delivers a constant current to the diode. In order to maximize the current modulation bandwidth, a shunt circuit was connected directly in parallel with the LD inside the DL100 laser head. Because the current source delivers a constant current to the laser, any current that passes through the shunt is at a the expense of the current going through the LD. The shunt nominally allows 2mA to
bypass the diode. This current changes in proportion to a signed voltage applied to the shunt circuit. The shunt circuit used in this servo was designed by Pierre Dubé of the NRC’s Frequency and Time Group, a schematic for this circuit has been included in Appendix C. The other circuits for this servo were designed by the author. Using current modulation, a wide bandwidth actuator was realized. A bode plot of its magnitude and phase response is shown in Figure 4.10.

![Bode Plot of DL100’s optical frequency response to current modulation. A) Amplitude response. B) Phase response.](image_url)

**Figure 4.10:** Bode Plot of DL100’s optical frequency response to current modulation. A) Amplitude response. B) Phase response.

The current response transfer function undergoes a $-90^\circ$ phase shift at 360kHz and a $180^\circ$ shift at 818kHz. The loop filter for the current control loop was designed to have a flat response up to its pole at 2.7kHz and then to roll off at 6db / octave. A zero in the filter causes its response to flatten out again at 723kHz. The result of this is that the loop transfer function doesn’t experience a $180^\circ$ phase shift for Fourier frequencies lower than 590kHz. This phase shift could be pushed to 700kHz by placing the zero of the loop filter at 360kHz as opposed to 723kHz as in the current design. This step has not yet been taken. A plot of the DL100 current response when convolved with the calculated loop filter response of the
current controller is shown in Figure 4.11.

![Bode plot](image)

**Figure 4.11:** Bode plot of the amplitude (A), and phase response (B), of the loop transfer function for DL100. The responses of the current control based actuator and the calculated response of the actuator combined with the loop filter are indicated.

A plot showing the effect of the PZT and CC feedback loops on the error signal observed on the FP is given in Figure 4.12. This figure shows that the servo loops have a significant effect on noise PSD as observed by the discriminator.

It must be emphasized that while interesting for observing the bandwidth of a servo loop, the error signal of a servo while locked to a cavity cannot be used to estimate the laser’s linewidth. The servo will work to minimize the effect of fluctuations in the cavity and noise in the detection system observed on the error signal by writing noise onto the laser. In order to accurately gauge the effectiveness of a servo loop, the laser must be evaluated with a third party discriminator which is in no way linked to the servo loop. For this experiment, the evaluation was done by heterodyning this laser system with the frequency doubled system and looking at the resulting linewidth. This evaluation will be discussed along with the overall cooling laser system design and performance in Section 4.5.
In order to achieve optimal Doppler laser cooling of the ion, the frequency of the cooling laser must be close to the half maximum point of the trapped ion’s cooling transition and red detuned relative to its line center [22]. If the laser drifts on the low frequency side of the $^{88}\text{Sr}^+$ S-P transition, the cooling efficiency will be reduced. Drifting onto the high frequency side will cause the laser light to heat the ion in the trap. In order to successfully load the ion, cool and maintain it in the trap, the frequency of the cooling laser must be known and set to a fraction of the $\sim$20MHz linewidth of the ion’s cooling transition. In order to facilitate this the laser can be stabilized on the $5s\,^2S_{1/2}(F'' = 2)\rightarrow 6p\,^2P_{1/2}(F' = 3)$ transition in $^{85}\text{Rb}$ which is located approximately 440MHz to the red of the cooling transition in $^{88}\text{Sr}^+$. In order to hold the laser frequency at the correct detuning relative to the $^{88}\text{Sr}^+$ line center, some of the radiation that is not sent to the ion is frequency shifted by $\sim$440MHz in an AOM and...
then the shifted frequency of the laser is locked onto the center of the $^{85}\text{Rb}$ absorption line described below.

### 4.4.1 Saturation Spectroscopy

The natural line width of the $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 3)$ in $^{85}\text{Rb}$ is 240 kHz [53, 54]. Unfortunately, a laser passing through a cell filled with Rb vapor is absorbed over the much larger Doppler broadened absorption profile such as the profile shown in Figure 4.13. This results from the fact that the Rb atoms are not stationary but are instead moving with a Maxwellian velocity distribution the width of which is determined by the atomic mass $m$ and the average temperature of the ensemble of atoms $T$. A laser directed along $z$ with frequency $\omega$ will be observed in the reference frame of an atom moving with a $z$-component of velocity $v_z$ as having frequency $\omega' = \omega(1 - v_z/c)$. A laser that is resonant with the atomic transition of a stationary atom at frequency $\omega_o$ would need to be tuned to the frequency $\omega_a = \omega_o(1 + v_z/c)$ to be absorbed by an atom moving with velocity $v_z$. The absorption intensity profile of a Doppler broadened spectral line follows the ensemble Maxwellian velocity distribution and can be expressed in terms of the laser frequency $\omega$ as [55]:

$$I(\omega) = I_o \exp \left[ - \left( \frac{c(\omega - \omega_o)}{\omega_o v_p} \right)^2 \right]. \quad (4.6)$$

Where $v_p$ is the most probable speed of the of the ensemble of atoms in the direction of the laser beam. The Doppler width of this distribution (FWHM) is:

$$\delta \omega_D = 2\sqrt{ln2}\omega_o v_p/c \quad (4.7)$$

$$= \left( \frac{\omega_o}{c} \right) \sqrt{8kTln2/m}. \quad (4.8)$$
The temperature of the Rb vapor used to lock the cooling laser is determined by the temperature of the cell walls \( \sim 100^\circ \text{C} \). For \(^{85}\text{Rb}\) the expected Doppler broadened linewidth for a single transition is 1.1 GHz (FWHM). The larger linewidth in Figure 4.13 results from the superposition of the two hyperfine transitions contained within the Doppler profile as well as broadening due to saturation of the transition.

Figure 4.13: \(^{85}\text{Rb}\) Doppler profile near the \( 5s\,^2S_{1/2}(F'' = 2) - 6p\,^2P_{1/2}(F' = 2, 3) \) transitions. The strongly saturated \( F'' = 2 \) and \( F'' = 3 \) lines are clearly visible along with a saturated crossover transition near the center of the broad Doppler profile.

One approach for observing the sub Doppler linewidth of a transition in an atomic vapor is the technique of saturation absorption spectroscopy [55]. The laser is directed through the gaseous sample with sufficient intensity to significantly depopulate the ground state of the transition, saturating the sample. As described above, the laser will only be absorbed by the group of atoms whose velocity with respect to the laser Doppler shifts them into resonance with the laser radiation. After passing through the cell the laser is retroreflected and
passes through the gas cell a second time. If the laser’s frequency does not correspond to the transition frequency of the atom in the rest frame, it will once again only be absorbed by those atoms in a velocity class that is Doppler shifted into resonance with the laser. Because the retroreflected (probe) beam is moving in the opposite direction to the saturating beam, atoms that were Doppler shifted into resonance with the saturating beam will be moving in the wrong direction to be shifted into resonance with the probe beam. Thus the saturation and probe beams will each interact with a different velocity group of atoms and the absorption will correspond to that of a single pass through a cell that was twice as long. The exception to this case occurs for the zero velocity group which experiences no Doppler shift with respect to either the saturation or the probe beams. Both beams will interact with the same group of atoms and because the saturating beam significantly depopulates the number of atoms in the ground state, the absorption of the signal beam is reduced. This results in a significant increase in the transmission through the cell when the laser is tuned into resonance with the zero velocity group. The two outer “Saturation Peaks” near the center of the Doppler broadened absorption profile in Figure 4.13 correspond to two hyperfine transitions in $^{85}\text{Rb}$ which were able to be resolved using this technique. The central peak is a crossover transition which occurs when the laser frequency is tuned exactly in between the frequencies of the two hyperfine transitions. The saturating beam interacts with atoms from each transition that are Doppler shifted by equal amounts in opposite directions bringing them into resonance with the laser. The probe beam then interacts with the same two groups of atoms, each of which is Doppler shifted in the opposite direction compared to their shift with respect to the counter propagating pump beam. However because the shifts from the lasers frequency to that of each transition are equal and opposite, both velocity classes also interact with the probe and a saturation peak is observed.
4.4.2 Third Harmonic Locking to Saturated Lineshape

To frequency stabilize a laser onto the center of an absorption line, it is necessary to generate a signed error signal with a magnitude that increases with the laser’s deviation from line center and whose sign indicates which side of line center the laser has drifted toward. The approach chosen here is to apply a small dither to the laser frequency and to demodulate the detected signal at an odd harmonic of the dither frequency. As will be shown below, the resulting waveform is proportional to an odd derivative of the saturated lineshape so it crosses through zero when the saturated lineshape is at its maximum corresponding to the line center. This error signal can then be used as the input to a feedback servo loop which controls the laser’s frequency to maintain the error signal at zero thus keeping the laser on the center of the saturated absorption line.

4.4.2.1 Modulation Broadening Theory

The effect of modulation depth and demodulation harmonic on an observed lineshape is treated by [56]. The expected signal that would be detected while modulating the lasers frequency and slowly sweeping the laser across a Lorentzian lineshape is given by

\[ g(t) = \pi^{-1} \frac{\frac{1}{2}H_{\frac{1}{2}}}{(\frac{1}{2}H_{\frac{1}{2}})^2 + (H_\delta + H_\omega \cos \omega t)^2}. \]  \hspace{1cm} (4.9)

Where \( H_{\frac{1}{2}} \) is the Full Width Half Maximum (FWHM) of the resonance lineshape, \( H_\omega \) is the modulation depth which has angular frequency \( \omega \) and \( H_\delta \) is defined as \( H_\delta = H_a - H_0 \) where \( H_0 \) is the center frequency of the resonance and \( H_a(t) \) describes a slow ramp in laser frequency. This detected signal can be expressed in terms of its Fourier components as follows
\[ g(t) = \frac{H_1}{2\pi} \sum_{n=0}^{\infty} a_n(H_1, H_\omega, H_\delta) \cos n\omega t \]  

(4.10)

where the Fourier amplitudes \( a_n \) are given by the integral

\[ a_n(H_1, H_\omega, H_\delta) = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} \frac{\cos n\omega t}{(H_1/2)^2 + (H_\delta + H_\omega \cos \omega t)^2} dt \]  

(4.11)

When the detected signal is demodulated with a phase sensitive detector [57] using the \( n^{th} \) harmonic of the modulation frequency \( \omega \) as its reference, a signal that is proportional to the \( n^{th} \) Fourier term in (4.10) is returned. Each Fourier term \( a_n \) is proportional to the \( n^{th} \) derivative of the lineshape. By demodulating at an odd harmonic, the resulting waveform crosses through zero at the Lorentzian’s center frequency and is antisymmetric about that point. When using the demodulated waveform as an error signal to lock a laser onto the line center frequency, phase sensitive detection at the first harmonic is often used. For applications in high resolution spectroscopy [58], third-harmonic detection is preferred as it is far less sensitive to any background which a Lorentzian feature may be sitting on.

If the absorption lineshape is expanded in a Taylor series, third-harmonic detection is only sensitive to the third derivative term in this expansion. Contributions from the constant, first and second derivative terms are not present in the demodulated waveform. This has the effect of largely suppressing any background that the Lorentizan lineshape may be sitting on such as the contribution from the Doppler profile surrounding a saturated absorption line in a saturation absorption spectrometer [55]. The sensitivity of first harmonic detection to the Doppler background can result in the error signal’s zero crossing having an offset relative to the waveform’s true line center unless additional steps are taken to subtract off this background [55]. The cost of demodulating at the third harmonic is that the \( a_3 \) fourier term is at least a factor of 2 smaller than the \( a_1 \) term (depending on the modulation depth
chosen), resulting in a smaller peak to peak error signal. This can be a problem if the signal to noise ratio of the detected signal is small. A plot of a Lorentzian lineshape along with the expected waveforms that would result from its phase sensitive detection at different harmonics is given in Figure 4.14.

![Figure 4.14](image)

**Figure 4.14:** Calculated result for a Lorentzian line shape (A) showing the result of first harmonic demodulation (B), Second harmonic demodulation (C) and third harmonic detection (D). In all cases the peak modulation depth $H_w$ was set to $0.9 H_{1/2}$. 

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4.4.3 Selection of Modulation Parameters

When using 3f detection, the choice of modulation depth has a significant effect on the amplitude of the resulting 3f spectra as well as the slope of the spectrum at the zero crossing for each line. To examine the effect of this, Equation (4.11) was evaluated for the case of third-harmonic detection as a function of the ratio of modulation depth $H_w$ to the absorption feature’s true Full Width Half Maximum ($H_{1/2}$). An analytic solution for (4.11) was given in [56] and has been included in Appendix A for reference. For third harmonic detection, the peak amplitude of the demodulated waveform is maximized when $H_w/H_{1/2} = 1.796$ and the slope at the zero crossing is maximized when $H_w/H_{1/2} = 0.818$.

![Figure 4.15: Effect of modulation depth on the demodulated wave form. (A) Amplitude of 3f demodulated waveform (proportional to $a_3$). (B) Frequency separation between peaks of 3f demodulated waveform. (C) Slope of 3f demodulated waveform at the lorentzian linecenter.](image-url)
4.4.4 Rb Saturation Spectroscopy Results

The saturation spectrometer used to obtain the spectra in Figure 4.13 is shown schematically in Figure 4.16. The laser passed through a 50% beam splitter and then double passed through the heated Rb cell. The beam which passes through the cell first was used to saturate the sample. It was then retroreflected and used to probe the group of atoms that were exposed to the counter propagating saturation beam.

![Figure 4.16: Rb Saturation Spectrometer](image)

The retroreflection was accomplished with a lens and mirror located at the focus of the lens. This arrangement insured that the probe beam was well mode matched to the saturation beam. A ND 0.5 filter was used to reduce the power of the probe beam prior to passing through the saturated Rb sample. This has been shown to improve the contrast of the saturated absorption spectra.

Prior to passing through the cell, the frequency of the laser was shifted by $\sim 440$MHz using a double passed Acousto-Optic Modulator (AOM). This modulator also applied a frequency modulation to the laser with a $\sim 9$MHz p-p frequency excursion that was used for 3f detection as described above. An example of the resulting 3f demodulated waveform is shown in Figure 4.17.
4.4.5 Rb Lock Servo Design

As stated earlier, the $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 3)$ transition in $^{85}$Rb is located $\sim$440MHz to the red of the $^{88}$Sr$^+$ cooling transition. To maintain the cooling laser at the correct detuning with respect to the $^{88}$Sr$^+$ cooling transition, some of the laser radiation which is not sent to the ion is frequency shifted down (red detuned with respect to the cooling transition) by 440MHz in an AOM. The shifted radiation is then locked onto the $^{85}$Rb transition which ensures that the unshifted part of the beam has the correct optical frequency for efficient cooling of the ion. The error signal used for this lock is the 3f demodulated waveform shown in Figure 4.17 where the $(F'' = 2) - (F' = 3)$ reference transition appears on the right hand side. The central zero crossing of a 3f demodulated waveform corresponds to the center of the saturated absorption line. A servo is used to hold the shifted laser’s frequency at this zero
crossing by using the 3f demodulated signal in a feedback loop. The feedback loop responds to this signed error signal by varying the mirror spacing of the FP that the laser is locked onto. Since the laser is locked onto the FP, it will follow any variation in the FP cavity spacing. This arrangement was chosen because the 3f lock servo only has a few hundred Hz of servo bandwidth and is therefore only used to make slow corrections to the laser absolute frequency through the FP. The sidelock to the FP has hundreds of kHz of useable bandwidth and is therefore used for laser line narrowing. Locking the shifted radiation onto the line center of the reference transition in this way ensures that the unshifted radiation remains resonant with the cooling transition. The absolute frequency of the cooling radiation that is directed onto the ion can be varied by adjusting the AOM detuning.

The 3f demodulated signal is produced by a Lock-In amplifier (Stanford Research SR830). The lock-in amplifier works by mixing the detected signal with the phase shifted third harmonic of the modulation signal and then low-pass filtering the result with a 1ms time constant. The demodulated signal after the low-pass filter is proportional to the $a_3$ term in (4.11). The servo’s loop filter was designed to behave as an integrator with a pole at zero Hz and a zero at 159Hz. This zero cancels the effect of the lock-in amplifier’s pole resulting in a loop transfer function that rolls off at a constant 6db / octave. The FP mirror spacing is adjusted by varying the voltage on a PZT actuator to which one of the FP mirrors is attached. The PZT is driven by a low noise High Voltage, (HV) (1000V max), amplifier which changes the FP spacing with a gain of 1800 MHz/V. With this gain, the 1.2 mV rms noise from the 3f lock servo circuit would cause significant broadening of the laser when locked to the FP. To prevent this, a filter was designed that allows 50% of the signal from the loop filter to reach the HV amplifier at zero Hz and only 5% for frequencies starting at $\sim$16 Hz. The effect of this circuit has been to reduce the electronic noise reaching the HV amplifier to $< 160 \mu$V
rms while allowing for sufficient gain at DC for the servo to correct for changes in cavity length resulting from changes in room temperature over the course of hours. The schematic of the 3f-lock servo is given in Appendix C as Figure C.6. The unity gain bandwidth of this servo was limited to \( \sim 1\text{kHz} \) since corrections in the error signal at frequencies approaching the modulation frequency will add noise to the laser.

### 4.5 Cooling Laser System Design

A diagram showing the optical layout of the new cooling laser system is shown in Figure 4.18. The laser is directed through two optical isolators (Conoptics 711c) which combined provide \( >60\text{dB} \) of reverse isolation. A \( f=65\text{cm} \) lens located between the two isolators when combined with a fiber collimation package (Thorlabs F220FC-A) couples the radiation into single mode fiber (Newfern S405) with \( > 50\% \) coupling efficiency. The desire to be able to strongly saturate the Rb reference as well as the effect of high losses in fused silica optics at 422nm motivated the allocation of 90\% of the laser’s radiation for stabilization onto the FP and spectroscopy of the Rb reference. A 90\% beamsplitter is used to divert part of the beam out of the path leading to the fiber coupler for laser stabilization and Rb spectroscopy. From the diverted beam 4\% of the radiation is reflected out using a glass plate and directed through the Fabry-Perot cavity described in Section 4.3.3.2. A \( f=70 \text{mm} \) lens before the FP brings the beam to a focus near the center of the cavity improving mode matching into the FP. The radiation emerging from the FP is detected with a low noise \( >1\text{MHz} \) bandwidth photodetector which was specially designed for this application and whose schematic has been included in Appendix C. A second identical detector which is visible above the second isolator in the diagram of the optical layout, detects light that is rejected by one of the isolator polarizing beamsplitters (PBS) and serves to monitor amplitude fluctuations in the laser’s intensity. The difference in the signals between these two detectors once balanced is proportional to
frequency excursions of the laser from the FP lock point as described in Section 4.3.3.2. The remainder of the light which was not deflected out to the FP by the glass plate is used for the Rb saturation spectrometer. The horizontally polarized beam first passes through a PBS and is then focused into an AOM (Crystal Technology model 3200-121). The 65 cm lens used for fiber coupling conveniently has a beam waist near the center of the AOM. The AOM shifts \( \sim 80\% \) of the radiation by \( \sim 200\text{MHz} \) and the shifted beam then passes through a \( f=75 \text{ mm lens} \) that has beam waists located both at the center of the AOM and on a retroreflecting mirror. The beam passes twice through a Quarter Wave Plate (QWP) located in between the lens and mirror. After the second pass through the QWP, beam polarization has changed from horizontal to vertical polarization. The beam passes a second time through the AOM which doubles the frequency shift to \( \sim 400\text{MHz} \) and causes the beam to be collinear with the beam that originally entered the AOM. The advantage of this “Double Passed AOM” configuration is that it greatly reduces the sensitivity of the frequency shifted beam to changes in pointing resulting from changes to the AOM’s shift frequency. The now vertically polarized beam is reflected out of the path of the incoming beam during its second pass through the polarizing beam splitter. An iris after this PBS blocks undesired diffraction orders from the AOM. The efficiency of the AOM was measured to be 80\% per pass but unfortunately the anti-reflection coating on this AOM was not designed to be used at 422nm, leading to large Fresnel loss. Also fluorescence is observed in the silica optics used in this system. Both of these factors combine with the 80\% per pass efficiency of the AOM to give an overall efficiency of \( \sim 25\% \) for frequency shifting the laser. The frequency shifted beam then passes through a telescope made from a \( f=75 \text{ mm lens} \) followed by an \( f=40 \text{ mm lens} \). This telescope reduces the beam diameter by a factor of 53\%. The beam passes through a 50\% beam splitter which directs half of the beam power into the Rb cell and the other half to a reference detector. The power of the saturating beam entering the heated Rb cell is typically \( \sim 330\mu\text{W} \) and its gaussian spot
size $\omega$ varies from 0.14mm to 0.25mm as it transverses the cell. After passing through the cell, the saturating beam is retroreflected by the combination of a lens and a mirror located at the focus of the lens. This combination ensures that the saturation and the retro reflected (probe) beams are well mode matched. An ND 0.5 filter located between the cell and the mirror reduces the intensity of the probe beam by a factor of $2 \times ND_{0.5}$. This significantly increases the contrast of the observed saturation peaks because the much weaker probe no longer has sufficient power to saturate the Rb vapor and will be strongly absorbed when tuned away from resonance with the saturated zero velocity group. After the signal beam passes through the Rb cell, it goes through the 50% beam splitter and then onto a signal detector. A beamsplitter was chosen for separating the saturation and probe beams instead of the PBS and QWP combination used for double passing the AOM so that the laser would have the same linear polarization state for each pass through the Rb vapor. Both the signal and reference beams from the saturation spectrometer are focused into separate channels of a balanced photo detector (New Focus Nirvana model 2007). The balanced photodetector removes most of the amplitude modulation (AM) imparted onto the beam during frequency modulation with the AOM. The lenses used to focus into the photodetectors reduce AM resulting from small beam pointing variations which occur while frequency modulating the AOM. The reference beam for this saturation spectrometer does not pass through the Rb cell and therefore cannot be used to suppress the Doppler profile surrounding the saturated Rb lines. The decision not to pass the reference beam through the Rb cell was largely motivated by space constraints on the optical breadboard as well as by the desire to reduce the system complexity. Fortunately as was shown in Section 4.4.2.1, by demodulating the detected signal at the third harmonic of the dither frequency applied to the AOM, curvature resulting from the Doppler background is suppressed up to and including terms that are second order in the frequency. This has largely removed offsets resulting from the Doppler background. A
Figure 4.18: Optical layout of cooling laser system

A photograph of the cooling laser system which has the same layout as the labeled diagram in Figure 4.18 is given in Figure 4.19. A picture showing the cooling laser system including its control electronics is shown in Figure 4.20.
4.6 Cooling Laser System Results

An accurate evaluation of the laser linewidth requires its frequency fluctuations to be observed using a discriminator that is not part of the laser frequency control feedback loop. The linewidth of the new laser was measured by combining radiation from this source with that of the older frequency doubled system on a photodetector. A heterodyne beat between these two lasers is observed using the photodetector when the optical frequencies of the two lasers are brought within a few hundred megahertz of one another. The resulting electrical signal was analyzed using an RF spectrum analyzer (Advantest R3261B). The width of the observed beat note’s Power Spectral Density is the convolution of the linewidths of the two
Figure 4.20: Photograph of cooling laser system. Major components are labeled as:
A) Laser and Stabilization Optics
B) DL100 Current Supply and Temperature Controller
C) Slow PZT Based Servo Controller
D) Current feedforward and PZT offset voltage controller
E) Fast Current Control servo controller
F) Rb 3f lock servo
G) DSP Lockin amplifier for Rb 3f Lock
H) Lockin amplifier at 2f for monitoring 3f lock
I) High Voltage amplifier for Fabry-Perot PZT
J) AOM Driver
K) Rb Oven Temperature Controller

lasers. The effect of the different servo controllers used to stabilize the new laser source on
the observed beat signal is shown in Figure 4.21. In all cases, the frequency of the doubled
source was not actively controlled. The linewidth of the frequency doubled system is known
to be \(~1\text{MHz}\) so the beat linewidths shown in these figures are dominated by that of the new laser source with the exception of Figure 4.21 D) which shows the linewidth when all of the new cooling laser’s servos were active. Figure 4.21 A) shows the beat between the free running DL100 and the frequency doubled system. Figure B) shows the effect of turning on the slow PZT based servo. Figure C) shows the effect of using both the slow PZT and fast current control (CC) servo to hold the DL110’s frequency at the lockpoint on the FP. Figure D) shows the effect of both the PZT and CC servos along with the 3f lock to the $^{85}\text{Rb}$ lockpoint. Especially for the case of the free running DL100, significant variation was seen in the beat signal. This variation was largely a sampling artifact caused by the effect of the beat signal either drifting with or against the direction of the spectrum analyzer frequency sweep. To accurately estimate the width of the beat spectrum, each case was measured a minimum of five times and in each instance the linewidth was determined as the separation between the points where the PSD first reached its half maximum on both low and high frequency sides. The average linewidths and their standard deviations are summarized in Table 4.1.

<table>
<thead>
<tr>
<th>Case</th>
<th>Average Linewidth</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Running</td>
<td>9.2MHz</td>
<td>2.82MHz</td>
</tr>
<tr>
<td>PZT</td>
<td>6.2MHz</td>
<td>1.18MHz</td>
</tr>
<tr>
<td>PZT + CC</td>
<td>3.5MHz</td>
<td>0.34MHz</td>
</tr>
<tr>
<td>PZT + CC + 3f</td>
<td>2.2MHz</td>
<td>0.34MHz</td>
</tr>
</tbody>
</table>

**Table 4.1:** Effect of different servo loops on the beat linewidth measured between the new cooling laser system and the frequency doubled system.

For cases where the current control (CC) based servo was not used, the cooling laser linewidth was dominated by large, slow, frequency excursions and the resulting beat lineshape appears
Figure 4.21: Heterodyne beat between the new cooling laser system and the old frequency doubled system showing the effect of various servos on the cooling lasers linewidth. A) Free running, B) Slow PZT servo lock to FP only, C) Slow PZT Lock and fast CC Lock to FP. D) Slow PZT and Fast CC locks to FP as well as 3f lock to $^{85}$Rb reference transition.

The resolution and video bandwidth for all plots was 1MHz except for D) which used a resolution bandwidth of 300kHz and Video Bandwidth of 100kHz. All plots were recorded using a 50ms sweep time.

rectangular. In the case where the PZT, CC and 3f servos were all used the resulting beat note has a Lorentzian lineshape. This case, which was shown in Figure 4.21 (D) is shown as an expanded plot in Figure 4.22. In this plot, only the points corresponding to the outer
envelope of the measured beat signal were included. A Lorentzian fit to this data gives a linewidth of 2.2MHz. A Lorentzian fit to a series of six successive measurements of this beat signal gives a mean linewidth of 2.0MHz ± 0.25MHz. This linewidth which is the convolution of the linewidths of the frequency doubled system and the new cooling lasers represents an upper limit for the linewidth of the new cooling laser system. These results show that frequency control servo loops are able to significantly reduce the linewidth of the cooling laser from > 9MHz to not more than 2MHz.

When used as the cooling laser source for the $^{88}$Sr$^+$ ion trap experiment, the stability of the absolute frequency of the laser is as important as its linewidth. An evaluation of the absolute frequency stability of the new laser system was carried out in connection with a frequency comb based measurement of the absolute transition frequencies of the hyperfine transitions in $^{85}$Rb near 422nm. This work, which is presented in Chapter 6 included a measurement of the long term stability of the cooling laser while locked onto the reference transition in $^{85}$Rb. An absolute measurement of the relative fluctuations of the cooling laser optical frequency over a 1200 second period while locked to the Rb reference transition is presented in Figure 4.23. The 1200, one second measurements of the laser frequency falls within a ±7.5kHz (1σ) error bar. For the 710 THz cooling laser, these fluctuations correspond to a variation of plus or minus one part in $10^{11}$ in 1200 seconds. This stability far exceeds the requirements for maintaining the correct detuning while cooling the $^{88}$Sr$^+$ ion and, as well be shown in Chapter 6 allowed for the development of $^{85}$Rb as a useful frequency standard at 422nm. This cooling laser system has remained locked for periods >12h. Results obtained using this new cooling laser system with the $^{88}$Sr$^+$ ion trap will be presented in the next chapter.
Figure 4.22: Enlarged view of the heterodyne beat between the new cooling laser and the old frequency doubled system shown in Figure 4.21 (D). The cooling laser was locked to the FP with both the slow and fast servos and also locked onto the $^{85}\text{Rb}$ reference transition. The points plotted correspond to the outer envelope of the beat spectrum shown in Figure 4.21 (D). A Lorentzian fit to this envelope gives a linewidth of 2.2MHz.
Figure 4.23: Result of an absolute measurement of the stability of the cooling laser measured at one point per second while locked onto the reference transition in $^{85}\text{Rb}$. Measurement was made using an optical frequency comb and includes the drift introduced by the AOM’s frequency synthesizer. The standard deviation of these measurements is 7.5kHz ($1\sigma$).
5 Use of 422nm Cooling Laser with the Trapped $^{88}\text{Sr}^+$ Ion

5.1 Introduction

With the optical frequency of the new cooling laser source stabilized onto a Fabry-Perot resonator and then locked onto a saturated absorption line in $^{85}\text{Rb}$, the system could be applied for laser cooling of $^{88}\text{Sr}^+$ ions. This chapter will describe a series of studies using the new cooling source with the $^{88}\text{Sr}^+$ ion. These studies demonstrate both the improved cooling and frequency control that has been achieved with this new source.

5.2 Measurement of $^{88}\text{Sr}^+$ Cooling Transition Linewidth

With part of the cooling laser’s radiation frequency shifted in an AOM and locked onto the $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 3)$ reference transition in $^{85}\text{Rb}$, the absolute frequency of the unshifted cooling radiation can be accurately set. It is possible to adjust the frequency of the cooling radiation by adjusting the AOM’s offset drive frequency. As stated earlier, the reference transition is approximately 440MHz to the red of the $^{88}\text{Sr}^+$ cooling transition. In order to study the $^{88}\text{Sr}^+$ cooling transition’s line shape, a ramp was applied to the AOM’s frequency. The effect of this ramp was to sweep the detuning between the fixed $^{85}\text{Rb}$ lock point and the cooling laser from $\sim$374MHz to $\sim$437MHz with a repetition rate of 0.01Hz.
During the sweep, both the ramp and a signal that is proportional to the $^{88}\text{Sr}^+$ fluorescence rate were recorded with a computer based data acquisition system. The cooling transition lineshape was measured for the case of different cooling laser powers, measured at the trap, of 14µW, 1.4µW and 450nW. A plot showing the results of these measurements is given in Figure 5.1.

![Figure 5.1](image-url)

**Figure 5.1:** Observed $^{88}\text{Sr}^+$ fluorescence as a function of the frequency shift of the cooling laser above the $^{85}\text{Rb}$ lock point. The fluorescence lineshapes for cooling laser intensities of 14µW, 1.4 µW and 450nw are indicated.

These plots show a steady increase in fluorescence as the cooling laser approached the S-P transition’s line center from the low frequency (red detuned) side. As discussed in Section 3.1.3, when the cooling laser is red detuned relative to the cooling transition, it has the effect of removing kinetic energy from the ion. When the laser is tuned onto the high frequency side of the cooling transition, it has the opposite effect, resulting in heating of the ion in the trap. This is observed in Figure 5.1 as a rapid decrease in fluorescence as the cooling
laser was tuned to higher frequency. The method used to ramp the laser’s frequency left some ambiguity as to the absolute frequency of the cooling laser along the ramp. The linearity of the ramp is expected to be adequate for making relative measurements of the transition linewidth to within $\sim 1$MHz, however the absolute frequency of the cooling transition’s line center should not be inferred from these results.

The halfwidth of the cooling transition is observed to increase significantly with increasing cooling laser power going from 14MHz at 450nW to 40MHz at 14$\mu$W. This is primarily the consequence of power broadening of the cooling transition. This along with other factors affecting the observed width of the cooling transition are discussed in [59]. This paper mentions that three-level interactions between the cooling and repump lasers, which act on a common $P_{1/2}$ state, may contribute to the broadening of the transition. A brief discussion of three level interactions is given in Section 5.5. Figure 5.2 shows an expanded view of the cooling transition when the ion is cooled with a laser intensity of 450nW at 422nm. This figure shows the overlapping result of multiple sweeps across the transition represented by different marker symbols for the data points belonging to each sweep. The measured transition halfwidth of 14MHz is indicated. The known natural linewidth for this transition is 11 MHz (HWHM) [60].

The fact that it is possible to repeatedly ramp the cooling laser over the $^{88}$Sr$^+$ cooling transition indicates a high degree of control over the cooling laser’s frequency. The majority of the uncertainty in the absolute frequency of the cooling laser during the sweeps shown in this section comes from uncertainty in the offset between the cooling laser and the $^{85}$Rb lock point. When used for laser cooling, the offset between the cooling laser and the Rb lock point is fixed and this shift can be measured and set at 416 MHz with a calibrated frequency counter. The result of this is that the frequency of the cooling laser can be accurately set at the half
Figure 5.2: Observed $^{88}\text{Sr}^+$ fluorescence as a function of the frequency shift of the cooling laser above the $^{85}\text{Rb}$ lock point. The cooling laser intensity, measured at the trap, was 450nW. The resulting 14MHz cooling transition half width is indicated.

maximum of the cooling transition prior to attempting to load the trap. When cooling with the old frequency doubled system, the uncertainty as to its absolute frequency added to the lengthy list of possible reasons for difficulty loading the trap. With the frequency of the new source locked, with a known offset from the saturated reference transition in $^{85}\text{Rb}$, the uncertainty as to the cooling laser frequency is removed. This greatly simplifies the process of loading $^{88}\text{Sr}^+$ into the ion trap, and as will be discussed in Section 5.4, allows from more effective cooling of the ion once loaded.
5.3 Fluorescence Detection of Ion’s Electronic State

The new cooling laser source was used for fluorescence detection of clock transition using the electron shelving technique described in Section 3.4. An example of quantum jumps observed in the ion’s 422nm fluorescence is shown in Figure 5.3. For this data, the cooling-probing-fluorescence detection cycle was repeated at a rate of 19Hz. During each cycle, the ion was cooled for 24ms and then probed for 910µs. Following the probe pulse, cooling resumed and the ion’s fluorescence was recorded. Each point in the plot corresponds to the measured fluorescence rate at the end of each cycle and the short interruptions in fluorescence indicate cycles where a quantum jump was observed.

Once excited into a state with natural lifetime $\tau$ the probability of decay per unit time is given by:

$$P(t, \tau) = \frac{1}{\tau} \exp \left( -\frac{t}{\tau} \right)$$

which can be observed as variation in the duration of the quantum jumps in Figure 5.3.

A histogram showing the measured distribution of excited state lifetimes observed from the duration of quantum jumps is shown in Figure 5.4. This distribution was determined from the 102 jumps observed during a 120s measurement, with the probe laser tuned near the line center of one Zeeman component of the $^{88}\text{Sr}^+$ clock transition. An exponential fit to this data is shown, which is found to be decreasing with a time constant of $\tau = (0.41 \pm 0.07)$s (1 $\sigma$). A detailed study of the lifetime of the $^{88}\text{Sr}^+$ ion’s $^2D_{5/2}$ state was carried out by (Letchumanan et al.) [61] who measured the decay times of 160 000 spontaneous decay events of the ion’s $^2D_{5/2}$ state. Their reported lifetime of 0.391s agrees to within 5% of the measured lifetime.
Figure 5.3: Observed single $^{88}\text{Sr}^+$ quantum jumps in 422nm fluorescence

reported above. The discrepancy between these two values is largely due to a statistical uncertainty resulting from the relatively small sample size of 102 decay events used in this measurement. An error was also introduced from the 19Hz cycle rate of the cooling-probing-fluorescence detection cycle. This detection rate limited the timing resolution for determining the duration of a quantum jumps to 53ms. The effect of this timing granularity becomes less significant as the sample size increases.

These results show that the new cooling laser system has been used successfully to determine the ion’s electronic state. Using this laser, the lifetime the clock transition was determined, which agreed with the accepted value to within 5% limited mostly by the finite sample size used in this measurement.
5.4 Measurement of Kinetic Temperature

In order to assess how well the new laser was cooling the ion, an experiment was performed to measure the ion’s kinetic temperature. As described in Section 3.1.1, the ion is confined in the three dimensional, harmonic pseudopotential produced by an RF Paul trap. Any kinetic energy imparted to the ion will result in periodic motion of the ion along each of the trap’s orthogonal axes at a secular frequency given by the depth of the potential well along that axis. When the probe laser excites a given clock transition, quantum jumps are observed at that transition line center or carrier frequency. Any periodic motion of the ion that has a component along the probe laser propagation direction \( k \) will result in a series of side bands on either side of the carrier displaced by multiples of the frequency of the harmonic motion. These sidebands can be resolved spectroscopically provided that their displacement from the carrier is much larger than the linewidth of the transition. An example of the radial sidebands
Figure 5.5: Measurement of a $^{88}\text{Sr}^+$ clock transition showing the radial sidebands at ±991 kHz from the carrier. The peak of a Gaussian fit to the carrier occurred at 37 quantum jumps and the average peak of gaussian fits to the two symmetric sidebands was 10 jumps.

surrounding the clock transition that were measured while the $^{88}\text{Sr}^+$ ion was cooled with the new cooling laser is shown in Figure 5.5. The ratio of the intensity of the carrier to that of the sidebands is related to the magnitude of the ion secular motion. The magnitude of this motion is in turn related to its kinetic temperature, so by measuring the ratio of the carrier to the sidebands, the ion’s kinetic temperature can be determined.

An expression for the ratio of the intensity of the first sideband $\sigma(1)$ to that of the carrier
\( \sigma(0) \) is given by (Urabe et al.) [62] as:

\[
\left[ \frac{\sigma(1)}{\sigma(0)} \right]_i = \left( 1 + \frac{\hbar \nu_i}{2k_B T} \right) \frac{I_1}{I_0} \left[ \frac{k^2 h}{4\pi^2 M \nu_i} \left( \frac{k_B T}{\hbar \nu_i} - \frac{1}{2} \right) \right]
\]  \hspace{1cm} (5.2)

where \( i = r, z \) are the trap orthogonal axis. This expression applies provided that the condition \( \hbar \nu_i \ll k_B T \) is satisfied. In the above expression, \( I_m \) is the modified Bessel function of order \( m \), \( h \) is Planck’s constant, \( k_B \) is Boltzmann’s constant, \( T \) is the effective temperature, \( \nu_i \) is the oscillation frequency of the secular motion of the ion, \( k_i \) is the component of the probe laser’s wave vector in the \( i \) direction, and \( M \) is the atomic mass. The projection of the laser’s \( k \) vector onto each axis of radial motion is \( 59^\circ \) with a conservative error estimate of \( \pm 10^\circ \). The component of the laser wavenumber \( (k = \frac{2\pi}{\lambda}) \) along the radial direction is \( k_r = k \cos(59^\circ) \). Seven scans of the carrier and the sideband of one of the clock transitions Zeeman components were performed. Numerical fits to this data gave the ratio of the average peak of the sideband to that of the carrier as being \( 0.35 \pm 0.05 \). Evaluating (5.2) for the temperature that corresponds to this ratio gives an ion temperature in the radial direction of \( T = 12.8 \text{mK} \ (\pm 14 \text{mK} / -5 \text{mK}) \). This result compares favorably to previously published results using the frequency doubled lasers of \( T = 39 \text{mK} \pm 10 \text{mK} \) [18] although it must be emphasized that this represents a preliminary measurement and a detailed evaluation of the ion trap’s temperature and heating rates was not carried out as part of this work. This measurement does illustrate that the new cooling laser has successfully cooled the ion to a very low temperature and it suggests that the ion’s temperature can now be brought well below the temperatures achieved with the frequency doubled cooling laser system.
5.5 Quantum Coherence Effects

As a final demonstration of the stability of the new cooling source, when used to cool the \( ^{88}\text{Sr}^+ \) ion, this section will briefly show some effects of three-level interactions involving the cooling and repump lasers. Up until now, the effect of the cooling laser was mostly discussed in terms of its action on the ion’s \( ^2S_{1/2} \) to \( ^2P_{1/2} \) transition which was treated as a two-level system. Reference was made to the fact that the 1:13 branching ratio from the P state to the \( ^2D_{3/2} \) level required an additional laser to pump the ion out of the D state, in order to maintain cooling. This ‘repump’ laser interaction with the D-P transition was also treated as though it were acting on a 2-level system. In reality, because both of these transitions share a common P state, they are not isolated 2-level systems, and quantum interference effects between the two transitions are possible. These effects were not observed in the NRC trap when the frequency doubled source was used for cooling. This may have partially resulted from the instability in the frequency doubled system as well as from the multimode operation of the 1092nm Nd\(^{3+}\) laser that was used to excite D-P transitions. Recently, the repump laser was replaced with a much more stable source (Koheras Adjustik Y10 PZT) which operates on a single longitudinal mode. With the ion loaded into the trap, and the new cooling laser source locked onto the \( ^{85}\text{Rb} \) reference transition, abrupt changes in the ion’s fluorescence were observed while tuning the frequency of the repump laser.

An atomic system consisting of two transitions that share a common upper state is referred to as a \( \Lambda \) configuration. The \( ^{88}\text{Sr}^+ \) ion’s S, P and D levels that are used for laser cooling are an example of a \( \Lambda \) configuration. One example of a quantum interference effect is the appearance of a non-connected dark state \( |\text{NC}\rangle \) which can occur when the \( \Lambda \) system is excited by two lasers simultaneously. In the case of the \( ^{88}\text{Sr}^+ \) ion, the non connected state is a superposition of the ground and metastable states that cannot be connected to the ion’s P state by the
atom-laser interaction Hamiltonian $V_L$, $\langle P_{1/2}|V_L|NC \rangle = 0$ [17]. Once in this state, the ion will no longer absorb or scatter photons until the state, whose lifetime is ideally determined by that of the metastable D state, collapses. In practice, this lifetime is reduced by the linewidths of the lasers involved. The appearance of a non-connected state depends on the relative detuning of the two laser sources from their respective transitions as well as the Rabi frequencies of each transition and the relative phase of the two lasers. Because of the strong dependence of the dark state on the quality of the interaction between the ion and the laser sources Lisowski et. al. [17], were able to use the contrast of this state as a sensitive means of detecting and minimizing micromotion in an ion trap. A full treatment of the line shape of three-level ions in Paul traps was developed by Schubert et al. [63], whose solution employs the density matrix formalism and allows for the numerical calculation of the expected lineshape. The lineshape for the $^{88}\text{Sr}^+$ system under the conditions used in this experiment was not calculated as part of this work. However, because it is now possible to maintain the cooling laser at a fixed frequency, it was possible to record the coherent interaction between these two sources by sweeping the repump laser’s frequency. A plot of the measured three-level cooling transition lineshape under various conditions is shown in Figure 5.6.

Using the new cooling laser system, it is possible to observe quantum coherence effects in the NRC $^{88}\text{Sr}^+$ trap which had not previously been seen. The stability and reproducibility of the cooling laser optical frequency allowed for a study of the S-P-D, Λ system where the frequency of the 1092-nm source was the only free parameter. The high contrast seen in the ‘Dark-State’ in Figure 5.6 (C) would not be visible if both the cooling and repump lasers sources were not highly stable or if there was significant residual micromotion in the ion trap.
Figure 5.6: Observed $^{88}\text{Sr}^+$ fluorescence as a function of the 1092nm ‘repump’ laser’s detuning. In all cases, the cooling laser was tuned 420MHz above the Rb reference transitions frequency. The lineshape was observed under various conditions A) $P_{1092} = 74\mu$W, $P_{422} = 10\mu$W, B) $P_{1092} = 37\mu$W, $P_{422} = 10\mu$W, C) $P_{1092} = 74\mu$W, $P_{422} = 3\mu$W, D) $P_{1092} = 7.4\mu$W, $P_{422} = 3\mu$W. In subfigures B through D a Lorentzian lineshape (solid) was superimposed on the measured data points (dots).
5.6 Measured Linewidth of Clock Transition

With the ion trapped and laser cooled, a series of measurements were performed in order to assess the minimum achievable width of the clock transition. In addition to the greatly improved cooling source described in this work, a number of major improvements have been made to the probe laser following the previous linewidth measurement in 2004 [24]. The 2004 measurement showed a clock transition linewidth of $(50 \pm 9)$ Hz and is the convolution of the laser linewidth (as observed in the reference frame of the ion moving in the trap) and the linewidth of the clock transition. The result of the most recent series of measurements is shown in Figure 5.7. The 5.4 Hz linewidth measured for the clock transition was reported at the IEEE International Frequency Control Symposium [25] in June of 2006. This measurement represents the narrowest linewidth to have been observed for the $^{88}\text{Sr}^+$ clock transition.

![Figure 5.7: Measured linewidth of clock transition.](image)

Pulse = 198.0 ms

FWHM = 5.1 Hz
6 Absolute Measurements of Optical Frequencies for Rb Transitions near 422nm

6.1 Introduction

At the NRC’s Institute for National Measurement Standards, the absolute frequencies of optical transitions can be directly measured using the institute’s optical frequency comb. The resulting measurement of an optical frequency is directly traceable to the institute’s realization of the second. This realization is in turn traceable to and participates in the worldwide ensemble of clocks from which the internationally agreed upon realization of the second is derived [23,64].

Research groups who do not have access to the the resources of a national metrology institute can still make absolute frequency measurements by stabilizing their laser source onto the reference transition of a well characterized atomic or molecular absorber. One of the best examples of this is the series of saturated absorption transitions in $^{127}$I$_2$ ranging from 514nm to 640nm [58]. The absolute frequencies of some of these lines have been carefully studied and agreed upon by researchers from around the world. Lasers stabilized onto these transitions are now routinely used as practical realizations of the meter [7]. The absolute frequency of a laser properly stabilized onto one of these transitions can be known to a fractional uncertainty of parts in $10^{-10}$ to $10^{-11}$ depending on the transition chosen.
While these transitions are very useful for researchers working in the visible, and a similar series of references are available in the infrared, there are very few well characterized references in the violet region of the spectrum. Transitions in $^{86}$Kr at 450nm and $^{198}$Hg 436nm from discharge lamps have been characterized with fractional uncertainties of $2 \times 10^{-8}$ and $5 \times 10^{-8}$ respectively [58]. In addition a $^{130}$Te$_2$ atlas has been developed for wavelengths extending from 420 nm to 500 nm with some transitions above 425 nm that have been measured to an accuracy of a few parts in $10^9$ [65].

Currently there are no known publications that report the absolute frequencies for transitions between hyperfine components of the $5s\,{}^2S_{1/2} - 6p\,{}^2P_{1/2}$ transition in Rb that are used to stabilize the cooling laser near 422nm. Values for the center of mass for the series of hyperfine transitions between the $5s\,{}^2S_{1/2}$ and $6p\,{}^2P_{1/2}$ manifolds have been given in recent review papers on Rb [53,54]. The most accurate report was for $^{85}$Rb with an associated uncertainty of 300MHz [66] as reported in [67]. This represents a fractional uncertainty of $4 \times 10^{-7}$. The center of mass of this transition for a mixture of $^{85}$Rb and $^{87}$Rb with their natural isotopic abundances was measured to within 855 MHz by Kratz [68]. While poorly characterized, the Rb transitions near 422nm are strong absorbers and heated cells containing Rb vapor are straightforward to realize experimentally. For this reason, the new cooling laser source along with the NRC optical frequency comb were used to make absolute measurements of the transition frequencies of the hyperfine lines in Rb near 422nm. These measurements, which are described in the following chapter, will define a new atomic reference frequency in the violet region of the spectrum.
6.2 Rb Spectra

The Rb cells used for these measurements contained a mixture of both $^{85}\text{Rb}$ and $^{87}\text{Rb}$ that is roughly equal to their natural abundances of 72% and 28% respectively. The nuclear spin for $^{85}\text{Rb}$ is $I = \frac{5}{2}$ and for $^{87}\text{Rb}$ it is $I = \frac{3}{2}$. Unlike $^{88}\text{Sr}^+$ which has a nuclear spin of zero, coupling between the nuclear spin in $^{85}\text{Rb}$ and $^{87}\text{Rb}$ and the electromagnetic field produced by their respective electrons results in hyperfine structure. In $^{85}\text{Rb}$, the allowed values for states of $J = \frac{1}{2}$ give a total angular momentum $F$ of $F = 2$ and $F = 3$. For $^{87}\text{Rb}$, the allowed components are $F = 1$ and $F = 2$. Transitions between hyperfine components satisfy the selection rules $\Delta F = 0, \pm 1$ and $F = 0 \rightarrow F = 0$. As shown in Figure 6.1 for each isotope, there are four possible optical transitions between hyperfine components of the $5s{}^2S_{1/2} (F'') - 6p{}^2P_{1/2} (F')$ levels, where $F''$ and $F'$ denote the ground and excited state respectively.

The hyperfine transitions in Rb near 422-nm were observed using the stabilized semiconductor laser and saturation absorption spectrometer described in Chapter 4. The 3f demodulated spectra observed while sweeping across all four $^2S_{1/2} - ^2P_{1/2}$ transitions for both isotopes is shown in Figure 6.2. The optimal phase for the signal used to demodulate the 3f spectrum varies with laser frequency and may not have been optimized over the full range of frequencies included in this plot. For this reason, the relative transition strengths should not be inferred from this figure. The designation for each line is given in Table 6.1.
Figure 6.2: 3f demodulated spectra of the transitions between hyperfine components of the $^2S_{1/2} - ^2P_{1/2}$ transition at 422nm in both $^{85}\text{Rb}$ and $^{87}\text{Rb}$. The state designations are summarized in Table 6.1

6.3 Procedure for Making Absolute Frequency Measurements

The Rb absorption spectra in two different cells designated Cell A and Cell B were measured. Both cells contained Rb with the natural isotopic ratio and no buffer gas. Cell A (Environmental Optical Sensors Inc., Part Number Rb2001-S) is 77 mm long with an outer diameter of 25mm and was purchased in 1997. Cell B (Triad Technologies, Part Number TT-RB-75-V-P) is 88mm long with an outside diameter of 25mm and was purchased in 2004. Comparison between two different cells that were manufactured at different times by different manufacturers reduces the likelihood of cell specific systematic shifts of the transition frequencies such as those resulting from impurities in the cell.

Each cell was placed in an oven that used the current passing through a resistive Nikrothal
Table 6.1: Spectral assignments for the $5s^2 \, ^2S_{1/2}(F'') - 6p^2 \, ^2P_{1/2}(F')$ transitions in both $^{85}\text{Rb}$ and $^{87}\text{Rb}$. Transitions between hyperfine components are labeled as $(F'', F')$

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<td>a</td>
<td>87</td>
<td>(2,1)</td>
</tr>
<tr>
<td>b</td>
<td>87</td>
<td>(2,1)+(2,2) cross-over dip</td>
</tr>
<tr>
<td>c</td>
<td>87</td>
<td>(2,2)</td>
</tr>
<tr>
<td>d</td>
<td>85</td>
<td>(3,2)</td>
</tr>
<tr>
<td>e</td>
<td>85</td>
<td>(3,2)+(3,3) cross-over dip</td>
</tr>
<tr>
<td>f</td>
<td>85</td>
<td>(3,3)</td>
</tr>
<tr>
<td>g</td>
<td>85</td>
<td>(2,2)</td>
</tr>
<tr>
<td>h</td>
<td>85</td>
<td>(2,2)+(2,3) cross-over dip</td>
</tr>
<tr>
<td>i</td>
<td>85</td>
<td>(2,3)</td>
</tr>
<tr>
<td>j</td>
<td>87</td>
<td>(1,1)</td>
</tr>
<tr>
<td>k</td>
<td>87</td>
<td>(1,1)+(1,2) cross-over dip</td>
</tr>
<tr>
<td>l</td>
<td>87</td>
<td>(1,2)</td>
</tr>
</tbody>
</table>

wire to heat the cell. The wire was helically wrapped along the length of the oven and then back on itself in such a way that the magnetic field produced by the current flowing through the wire in one direction would be approximately canceled by the current flowing through the wire in the opposite direction. Three layers of 0.005 inch thick nickel sheeting wrapped around the cell provided partial shielding from magnetic fields. The fields inside the cell were measured with a gauss meter (Bell model G10) and did not exceed 250mG in the longitudinal or transverse directions in either cell. The contribution from the heating wire was measured to be below the 10mG sensitivity of the instrument. The temperature of the oven was maintained at $107^\circ \pm 1^\circ$ Celsius. The vapor pressure of Rb in the cell is determined by the vapor pressure of the liquid Rb which condenses at the coldest point in the cell. The temperature of this coldest point (commonly referred to as a cold finger) was determined by the temperature of a metal bracket that was attached to the Cell and kept out of direct thermal contact with the oven during the assembly process. It was not considered to be possible to make an
accurate measurement of the temperature of the cold finger in either of these cells however
it is expected that this temperature will be at most a few degrees less than that of the oven.

The laser source and Rb spectrometer used for this experiment has been described in Chap-

ter 4. For this study, the Gaussian beam spot size of the beam passing through the cell varied
from $\omega = 0.14\text{mm}$ to $\omega = 0.25\text{mm}$ along the length of the cell. For these measurements, the
saturation power was typically $330\mu\text{W} \pm 30\mu\text{W}$. The frequency of the radiation going to the
spectrometer was shifted by $-380\text{MHz}$ using a double passed AOM and modulated with an
excursion amplitude of $H_\omega = 4.5\text{ MHz}$ at a rate of $12.8\text{ kHz}$. An example of the resulting $3\text{f}$
demodulated spectra for lines g-i is shown in Figure 6.3.

![Figure 6.3: 3f demodulated spectrum of lines g,h and i](image)

Absolute measurements of the optical transition frequencies of the Rb lines were made by
stabilizing the external cavity diode laser onto a given line and then measuring the optical frequency of the part of the beam that did not pass through the AOM with the optical frequency comb. Unfortunately, the range of wavelengths spanned by this comb ranges from 450 nm to 1100 nm and does not extend far enough into the violet to measure the Rb transition frequencies at 422 nm directly. In order to make this measurement, a modified version of the original 844 nm frequency doubled laser source was employed. The heterodyne beat note between the doubled system and the new cooling laser while locked to a given Rb line was counted with a frequency counter.

![Figure 6.4: Arrangement for making optical frequency comb measurements of Rb lines at 422 nm. The 422 nm laser once stabilized onto a Rb line is heterodyned with the frequency doubled 844 nm radiation on an Avalanche Photo Detector (APD). Approximately 10% of the 844 nm radiation is simultaneously sent to the optical frequency comb for measurement.](image-url)

The optical frequency of the 844 nm source used to produce the doubled radiation was then directly measured with the optical frequency comb. The expression for the absolute frequency of a given Rb line while measured using this approach is:

$$f_{Rb} = 2(nf_{rep} \pm f_0 \pm f_b) + f_{RF} - 2f_{AOM}. \quad (6.1)$$

Where $f_{Rb}$ is the absolute frequency of the measured Rb line, $n$ is the comb order, $f_{rep}$ is the
ultrafast laser’s repetition rate and $f_0$ results from the carrier envelope offset as described in Section 3.3. The beat frequency $f_b$ is the measured offset between the 844-nm source and the nearest comb element, $f_{RF}$ is the beat frequency between the doubled system and the cooling laser and $f_{AOM}$ is the frequency shift imparted by each pass through the AOM. The frequency comb was operated by Dr. John Bernard, of the NRC Frequency and Time Group, who measured $f_{rep}$, $f_0$, and $f_b$ and determined their correct signs. The sign of $f_{RF}$ was determined by moving the frequency of the 844nm source in a known direction and observing the change in $f_{RF}$. The sign of $f_{AOM}$ is known from the arrangement of the double passed AOM optics. In order to determine the comb order, $n$, the frequency of the 844 nm source was also measured with a calibrated wavemeter (Burleigh WA1500) which is considered to be accurate to within ±150 MHz. All five frequencies in (6.1) were counted simultaneously in one second intervals with counters that were directly referenced to a hydrogen maser. The frequency offset between this maser and the ensemble of clocks that define the internationally agreed upon SI second, was recorded by the NRC Time Group, and applied as a correction to the measured absolute Rb transition frequencies. A given measurement of a Rb line absolute transition frequency typically consisted of 120, one second samples. Each sample involved the measurement of five different frequencies which were used along with equation (6.1) to determine the absolute frequency of the Rb line for that sample. The 120 samples were then averaged to determine an average transition frequency for that measurement. An example of the data from one, 120s, comb measurement of line (k) is shown in Figure 6.5. An example of the observed spectrum while scanning across lines g-i is shown in Figure 6.3. This spectrum was acquired using a 1ms time constant on the lockin amplifier and averaged over four traces.
Figure 6.5: Example of 120, 1 second measurements of the absolute frequency of Rb line (k). The mean frequency of this measurement is 710 964 944 941.7 [kHz] ± 2.9 [kHz] and is indicated by the dotted line.

6.4 Results

Lines (a) through (l) were each measured on three separate days using Cell B and two separate days using Cell A. The mean frequency for each transition line was determined by averaging all measurements from both cells and all measurement days. The associated uncertainty indicates the (one sigma) resetability of the of the absolute frequency of the laser while locking to each line. The results of these measurements and their associated uncertainties are given in Table 6.2.

As indicated at the beginning of this chapter, there are very few well characterized reference transitions in the violet region of the spectrum. In a 2006 review paper on Rb spectra, Sansonetti [53] referenced a 1949 paper by Kratz [68] to give the transition frequency for the Rb $5s^2S_{1/2} - 6p^2P_{1/2}$ transition. The measurement by Kratz used a 21 foot grating spectrometer and states the frequency of this transition as 710 963 510 MHz ± 840 MHz. The
<table>
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**Table 6.2:** Measured absolute frequencies for the $5s^2S_{1/2}(F'') - 6p^2P_{1/2}(F')$ transitions in both $^{85}$Rb and $^{87}$Rb.

The review paper goes on to list this transition frequency for the specific case of $^{85}$Rb as being 710 960 245 MHz ±300 MHz. This measurement was made by Johansson in 1961 [66]. Currently there are no other known publications that give the transition frequencies for the $5s^2S_{1/2}-6p^2P_{1/2}$ transition in Rb to a comparable or higher accuracy than those mentioned. A plot showing the measured value and uncertainties reported by other authors along with the results from this work is shown in Figure 6.6. The statistical uncertainties for the lines measured in this work were all less than ±30 kHz and for most lines were well under ±25 kHz. This represents at least a 4 orders of magnitude improvement in accuracy over previously reported results and the only absolute measurement to resolve the hyperfine structure of the $5s^2S_{1/2}-6p^2P_{1/2}$ transition in Rb.

The splitting between hyperfine components in the ground $5^2S_{1/2}$ and excited $6^2P_{1/2}$ states of $^{85}$Rb and $^{87}$Rb have been studied by many authors. These results have been reviewed by
Figure 6.6: Comparison of this work to other known measurements of the $^2S_{1/2} - ^2P_{1/2}$ transitions in Rb. The results of both Kratz [68] (K1949) and Johansson [66] (J1961) are indicated with error bars. Neither author was able to resolve hyperfine structure at 422nm. The $\lesssim 25$ kHz error bars for this work are not visible on the scale shown. The line designations are given in Table 6.1.

Arimondo et al. [69] who reports the hyperfine splitting for a given isotope and state in terms of its respective A coefficient. The accuracy to which the hyperfine splittings are known allows them to be used to test the results presented in this chapter. In the present work, the optical frequencies of four transitions are reported for each isotope. From Figure 6.1, it can be seen that these consist of two pairs, each with transitions that differ by the hyperfine spacing of either the ground or excited states. The energy difference between the transitions in each pair was calculated and compared to the expected splitting evaluated using the A coefficients given in [69] and equation (2.22). The results of these calculations and their associated uncertainties are summarized in Table 6.3.

These results show that most of the absolute transition frequencies reported in Table 6.2 have the expected energy spacing within the stated uncertainty. The exception to this are transi-
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<th>Calculated Value [kHz]</th>
<th>Difference [kHz]</th>
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<td>$^2\text{S}_{1/2}$ (g-d)</td>
<td>3 035 699 ± 38</td>
<td>3 035 732 ± 0.10</td>
<td>−34 ± 38</td>
</tr>
<tr>
<td></td>
<td>$^2\text{S}_{1/2}$ (i-f)</td>
<td>3 035 739 ± 17</td>
<td>3 035 732 ± 0.10</td>
<td>7.1 ± 17</td>
</tr>
<tr>
<td></td>
<td>$^2\text{P}_{1/2}$ (f-d)</td>
<td>117 372 ± 32</td>
<td>117 339 ± 100</td>
<td>33 ± 105</td>
</tr>
<tr>
<td></td>
<td>$^2\text{P}_{1/2}$ (i-g)</td>
<td>117 413 ± 27</td>
<td>117 339 ± 100</td>
<td>74 ± 104</td>
</tr>
<tr>
<td>$^87\text{Rb}$</td>
<td>$^2\text{S}_{1/2}$ (j-a)</td>
<td>6 834 625 ± 26</td>
<td>6 834 683 ± 0.10</td>
<td>−58 ± 26</td>
</tr>
<tr>
<td></td>
<td>$^2\text{S}_{1/2}$ (l-c)</td>
<td>6 834 676 ± 13</td>
<td>6 834 683 ± 0.10</td>
<td>−6.2 ± 13</td>
</tr>
<tr>
<td></td>
<td>$^2\text{P}_{1/2}$ (c-a)</td>
<td>265 142 ± 18</td>
<td>265 126 ± 10</td>
<td>16 ± 20</td>
</tr>
<tr>
<td></td>
<td>$^2\text{P}_{1/2}$ (l-j)</td>
<td>265 194 ± 23</td>
<td>265 126 ± 10</td>
<td>68 ± 25</td>
</tr>
</tbody>
</table>

**Table 6.3:** Comparison of the known splitting between hyperfine components of the $5s\ ^2\text{S}_{1/2}$ and $6p\ ^2\text{P}_{1/2}$ states in $^85\text{Rb}$ and $^87\text{Rb}$ to the splittings measured in this work.

The crossover transitions observed in a saturated absorption spectrum should occur symmetrically between pairs of transitions that share a common Doppler profile. The measured frequency of each crossover transition reported in this work is compared to the difference of the measured Doppler free transition frequencies of the lines participating in the crossover transition in Table 6.4.

This result shows that to within the stated uncertainties, the measured transition frequencies of the crossover transitions is equal to the average of the frequencies of the two transitions participating in the crossover. This combined with the close agreement of the measured fre-
<table>
<thead>
<tr>
<th>Line</th>
<th>Measured Value [kHz]</th>
<th>Lines in Avg.</th>
<th>Difference [kHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>710 959 306 825 ± 22</td>
<td>(d, f)</td>
<td>−19 ± 39</td>
</tr>
<tr>
<td>h</td>
<td>710 962 342 552 ± 12</td>
<td>(g, i)</td>
<td>−12 ± 29</td>
</tr>
<tr>
<td>b</td>
<td>710 958 110 272 ± 7.7</td>
<td>(c, a)</td>
<td>14 ± 19</td>
</tr>
<tr>
<td>k</td>
<td>710 964 944 934 ± 11</td>
<td>(j, 1)</td>
<td>−2.9 ± 25</td>
</tr>
</tbody>
</table>

Table 6.4: Comparison between the measured frequency of each crossover transition and the average of the measured frequencies of the transitions participating in that crossover transition. The difference between the measured crossover frequency and this average along with the associated uncertainty is given.

quency intervals between lines with the intervals predicted from the known hyperfine splitting gives a strong indication that the transition frequencies stated in this section are consistent.

6.4.1 Systematic Shifts

The development of any frequency standard requires not only an accurate determination of the standard’s frequency but also a clear specification of the conditions under which that transition frequency can be realized. To provide this, it is necessary to evaluate the sensitivity of the standard to any systematic effects that may shift the absolute frequency of the reference. A detailed evaluation of the sensitivities of the Rb reference at 422nm has not yet been carried out. This section presents preliminary results.

6.4.2 Effect of Magnetic Fields

The effect of an external magnetic field, on a Rb atom, is to cause a given hyperfine component to split into several $M_F$ sub-components. This is similar to the Zeeman splitting of the
fine structure levels observed in the $^{88}\text{Sr}^+$ clock transition. The present series of measurements lacked the necessary sensitivity to resolve the $\lesssim 2\text{MHz} / \text{Gauss}$ splitting that exists in the $< 250mG$ magnetic field present inside the oven. To first order, the $M_F$ components split symmetrically about the field free line center of their respective hyperfine component. Provided that there is no optical pumping, the center of mass of transitions to the various $M_F$ components of a hyperfine level will correspond to the component’s field free line center. In order to prevent optical pumping in this experiment, both the probe and cooling beams in the saturation spectrometer were linearly polarized. This polarization state is assured by the linear polarizer that is used as part of the double passed AOM setup described in Section 4.5.

### 6.4.3 Pressure Shift

The absence of a well defined cold finger on the Rb cells used for these measurements complicates the accurate determination of the Rb pressure inside each cell. This pressure is determined by the temperature of the coldest point in the cell. The temperature of the oven that the cell sits in can be used to set an upper limit for this pressure. For all measurements described in this chapter, the temperature of the oven was maintained at $107^\circ C \pm 1^\circ C$. The vapor pressure for Rb is given as a function of temperature in the CRC Handbook [70] as:

$$\log\left(\frac{p}{\text{Pa}}\right) = 5.006 + A + \frac{B}{T}.$$  \hspace{1cm} (6.2)

Where $p$ is the Rb vapor pressure in Pascals, $T$ is the temperature of the coldest point in the cell in Kelvin and for Rb in the liquid phase $A = 4.312$ and $B = -4040$. This equation is considered to be accurate to within better than 5%. For the cells used in this measurement, the upper limit on the Rb pressure is 59 mPa.

Although a study of the pressure shift sensitivity of the Rb lines near 422nm has not yet been
carried out, it is possible to get a sense for the order of magnitude of the shifts involved by looking at the sensitivities of other absorbers. Table 6.5 lists transitions in $^{130}$Te$_2$, $^{127}$I$_2$ and $^{87}$Rb which have been developed as frequency references. The sensitivity of each transitions frequency to changes in pressure along with the nominal vapor pressure used with each reference is indicated.

<table>
<thead>
<tr>
<th>Species</th>
<th>Transition</th>
<th>Pressure Shift [kHz/Pa]</th>
<th>Pressure</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{130}$Te$_2$</td>
<td>No. 1326(e$_3$)</td>
<td>-8</td>
<td>91 Pa</td>
<td>[71]</td>
</tr>
<tr>
<td>$^{127}$I$_2$</td>
<td>11 – 5 R(127)</td>
<td>-10</td>
<td>16 Pa</td>
<td>[72]</td>
</tr>
<tr>
<td>$^{87}$Rb</td>
<td>5S$<em>{1/2}$-5D$</em>{5/2}$ (2-photon)</td>
<td>-110</td>
<td>8.7 mPa</td>
<td>[73]</td>
</tr>
</tbody>
</table>

**Table 6.5:** Reported pressure shifts for reference transitions in $^{130}$Te$_2$, $^{127}$I$_2$ and $^{87}$Rb.

This data suggests that pressure shifts are typically on the order of -100 kHz / Pa or less. For the Rb cells used in this work, a -100 kHz / Pa sensitivity would result in a $\sim$ 6 kHz shift. While the pressure shift sensitivity will vary for different transitions and atomic or molecular species, it is reasonable to expect that the frequency shift for the Rb transitions near 422nm would be on the order of several kHz. Furthermore as the temperature of the cell is regulated it should be possible to reduce pressure related fluctuations to far less than the measured resetability of this system.

### 6.4.4 Power Shift

Laser radiation has the effect of broadening as well as shifting atomic transitions. Power broadening [55] occurs at laser intensities that are high enough to significantly depopulate the ground state of a transition. The reduced absorption near the atomic resonance causes a relative increase in the absorption in the wings of the transition. The result of this is an increase in linewidth. Light induced shifts in an atomic transition can result from the
Stark effect which describes the polarization of the positively charged nucleus and negatively charged electrons in the presence of an electrical field [10]. This polarization causes the center of mass of the electrons to move away from that of the nucleus. This causes a shift in the atomic energy levels that is proportional to the square of the field strength.

An example of power broadening was seen in Section 5.2 where the width of the $^{88}\text{Sr}^+$ cooling transition showed a strong dependence on the intensity of the cooling radiation. Broadening of the absorption profile of a transition can cause the profile to partially overlap with that of a nearby transition. The resulting asymmetry in the observed lineshape [23] can cause an offset in the measured transition frequency. This can be particularly severe when an absorber has a dense spectrum of closely spaced lines. The spacing between the saturated absorption lines in Rb measured in this work was never less than 59 MHz. As this spacing is much larger than the measured transition linewidth of $<5\text{MHz}$, powerbroadening was not expected to play a significant roll in the determination of line center.

To study the sensitivity of the measured Rb transition frequencies presented in this work to variations in laser intensity, the transition frequency of line (i) was measured as a function of the intensity of the pump beam. For each measurement the 3f demodulation phase was reoptimized and the laser was relocked onto the Rb line. A plot of the result of the power shift study for line (i) is shown in Figure 6.7. The 9 kHz error bars shown in this plot are dictated by the resetability for measurements of line (i) as given in Table 6.2. The pump power normally used to saturate the Rb sample is 330 $\mu$W. In this study, pump intensities ranging from 93 $\mu$W to 359 $\mu$W were used and the resulting absolute frequency of line (i) was measured. These measurements show no statistically significant dependence of the Rb transition frequency on the intensity of the pump beam within the error bars shown. While
Figure 6.7: Rb line (i) absolute frequency measured as a function of pump power. The error bars are ± 9 kHz which were determined from the measured resetability of line (i) given in Table 6.2.

A more detailed study involving measurements of more of the Rb lines is still needed, this preliminary result suggests that pump intensity does not significantly effect the measured transition frequency of a given line.

6.4.5 Modulation Shift

Section 4.4.3 showed the dependence of the amplitude and broadening of a 3f demodulated signal on the amplitude of the laser modulation $H_\omega$. Maximizing the observed 3f signal requires the modulation amplitude to be nearly double the FWHM of the saturated absorption line. The trade off of using a large modulation depth is that the determination of the transition linecenter includes a large contribution from the wings surrounding the saturated lineshape. Offsets resulting from third order curvature in the surrounding Doppler profile as well as contributions from neighboring lines become more significant as the modulation amplitude
increases. The sensitivity of the measured transition frequency of line (i) to the choice of modulation amplitude was studied for modulation amplitudes ranging from $H_\omega = 2.69\text{MHz}$ to $H_\omega = 8.05\text{MHz}$. The nominal modulation amplitude that is used with this spectrometer is $H_\omega = 4.47\text{MHz}$. The 3f demodulation phase was reoptimized and the laser was relocked onto line (i) for each modulation setting used in this study. The result of this study is shown in Figure 6.8.

![Figure 6.8: Rb line (i) absolute frequency measured as a function of modulation amplitude. Error bars are ± 9 kHz dictated by measured resetability of line (i) see Table 6.2](image)

The modulation shift observed from this data is $(−10.3 \pm 5.3)$ kHz / MHz. This preliminary result suggests that the sensitivity of the measured linecenter to the choice of modulation depth is relatively small. Experimentally, it is not difficult to control the modulation amplitude to within a fraction of a MHz. This suggests that the contribution from the choice of modulation depth to the uncertainty in the measured linecenter can be maintained at a level
that is much smaller than the overall uncertainty in the measurement of the transition’s line-center. Additional studies are needed to verify that the sensitivity of the other Rb transitions reported in this work are consistent with that of line (i). This preliminary result suggests that the modulation shifts should be small and easily controlled.
7 Conclusions

A new cooling laser system has been developed and is now routinely used for cooling and fluorescence detection of a $^{88}\text{Sr}^+$ single ion optical frequency standard. The linewidth of this laser was narrowed from 9MHz to not more than 2MHz by a high bandwidth (590 kHz) frequency control servo. The laser’s absolute frequency was controlled through active stabilization onto the saturated $5s^2S_{1/2}(F'' = 2) - 6p^2P_{1/2}(F' = 3)$ transition in neutral $^{85}\text{Rb}$.

Using this source, a single $^{88}\text{Sr}^+$ ion was trapped and laser cooled for a period exceeding 12 hours without either interruption of the cooling radiation or loss of the ion from the trap. An evaluation of this laser’s absolute frequency stability showed that fluctuations were limited to $\pm 7.5$ kHz ($1\sigma$) over a twenty minute period. Studies with the ion resolved the cooling transition half width as 14 MHz which is close to the 11 MHz natural halfwidth of the transition. The laser was used for fluorescence detection of the ion electronic state and analysis of the resulting quantum jump data gave the ion’s $5s^2S_{1/2} - 4d^2D_{5/2}$ transition lifetime as $\tau = (0.41 \pm 0.07)\text{s} (1\sigma)$ which agrees to within 5% of the accepted value. A temperature measurement of the ion while cooled with this source indicated that the ion had reached a temperature of $T = 12.8\text{mK} (+14\text{mK} - 5\text{mK}) (1\sigma)$, which is an improvement over the $T = 39\text{mK} \pm 10\text{mK} \ [18]$ results published using the previous cooling laser system. The stability of this source enabled three-level interactions in the ion’s S-P-D, $\Lambda$, system to be observed for the first time in the NRC $^{88}\text{Sr}^+$ ion trap experiment. While cooled with this source,
the clock transition linewidth was measured as 5.1 Hz (FWHM) representing the narrowest linewidth ever observed for the $^{88}\text{Sr}^+$ clock transition.

Measurements of the absolute frequency of this new laser source while stabilized onto the $5s^2S_{1/2} - 6p^2P_{1/2}$ transitions in both $^{85}\text{Rb}$ and $^{87}\text{Rb}$ allowed for an accurate measurement of the absolute frequency of this transition for both isotopes. This work represents the only known measurements to report absolute optical frequencies for the hyperfine transitions near 422 nm in either isotope. While consistent with earlier reports, the absolute optical frequencies with their associated $<30$ kHz resetability presented in this work represent an improvement of at least four orders of magnitude in absolute accuracy compared to previous reports. These measurements will define a new atomic frequency reference in a region of the spectrum where there are currently very few well characterized standards. A preliminary study of the sensitivity of these transitions to systematic shifts shows that the sensitivities are small and should be easily controlled.

The performance of the $^{88}\text{Sr}^+$ standard is currently considered to be limited by the inability to detect and minimize residual micromotion of the ion in directions orthogonal to that of the cooling radiation. The result of this is that the ion can undergo significant driven motion in the RF field of the Paul trap causing heating of the ion as well as a second order Doppler shift in the $^{88}\text{Sr}^+$ clock transition [18]. The next major step for the NRC $^{88}\text{Sr}^+$ ion trap program will be the completion of a next generation trap with optical access for a three beam cooling geometry. Using this geometry and the cooling laser that was developed as part of this work, it will be possible to detect micromotion in all three directions by observing the correlation between the ion fluorescence and the RF field in the Paul trap [74]. By applying electrical potentials to sets of orthogonal electrodes near the trap, it will be possible to minimize the
observed photon correlation by positioning the ion near the center of the trap. When this has been achieved, the micromotion of the ion will have been minimized.

The high accuracy and small statistical errors measured for the absolute frequencies of the Rb $5s^2S_{1/2} - 6p^2P_{1/2}$ transition reported in this thesis show that this system has great potential as an atomic reference. For this reference to reach its full potential, a detailed study of its sensitivity to systematic shifts will need to be conducted. This should include measurements of the effect of pressure, pump power and modulation amplitude for a number of different lines in the series. After this work has been completed and published it is expected that these reference transitions will be useful for researchers in a number of different labs.
Bibliography


A Analytic Solution for \(n^{th}\) Harmonic Demodulated Lineshape

As described in Section 4.4.2.1 the transmitted intensity of a laser modulated at frequency \(\omega\) with peak frequency excursion \(H_\omega\) can be expanded in a Fourier series as:

\[
 a_n(H_{\frac{1}{2}}, H_\omega, H_\delta) = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} \cos n\omega t \left(\frac{1}{2}H_{\frac{1}{2}}\right)^2 + \left(H_\delta + H_\omega \cos \omega t\right)^2 dt.
\] (A.1)

Where \(H_\delta\) is the laser’s detuning from the Lorentzian line center and \(H_{1/2}\) is the FWHM linewidth of the normalized Lorentzian. The amplitude of the \(n^{th}\) Fourier term was solved analytically in [56] and has been included here (with corrections) for reference.

First the dimensionless parameters \(\alpha\) and \(\beta\) are defined where:

\[
 \alpha = \left(\frac{H_\delta}{H_\omega}\right), \quad -\infty < \alpha < \infty \quad (A.2)
\]

\[
 \beta = \left(\frac{1}{2}H_{\frac{1}{2}}/H_\omega\right), \quad 0 < \beta < \infty \quad (A.3)
\]

Also auxiliary variables \(\gamma, u, r\) and \(\phi\) are defined as:
\[ \gamma = 1 + \beta^2 + \alpha^2 \quad (A.4) \]

\[ u = \gamma + [\gamma^2 - 4\alpha^2]^{\frac{1}{2}} , \quad 2 < u < \infty \quad (A.5) \]

\[ r = [u - 1 - u^\frac{1}{2}(u - 2)^{\frac{1}{2}}]^{\frac{1}{2}}, \quad 0 < r < 1 \quad (A.6) \]

\[ \phi = \arccos\{-\sqrt{2}\alpha/u^\frac{1}{2}\}, \quad 0 < \phi < \pi. \quad (A.7) \]

In terms of these variables the solution to A.1 for any \( n \) is:

\[ a_n(H_{\frac{1}{2}}, H_\omega, H_\delta) = \left( \frac{2}{H_\omega} \right)^2 \left\{ \frac{[r^{n-1} + r^{-(n-1)}] \sin(n+1)\phi - [r^{n+1} + r^{-(n+1)}] \sin(n-1)\phi}{[r^{-1} - r][r^2 + r^{-2} - 2\cos2\phi] \sin\phi} + I_n \right\} \quad (A.8) \]

where

\[ I_0 = I_1 = 0 \quad (A.9) \]

\[ I_2 = 1 \quad (A.10) \]

\[ I_3 = \frac{1}{z} + \frac{1}{z^*} + z + z^* \quad (A.11) \]

\[ I_n = \left\{ \left( \frac{d}{dx} \right)^{n-2} \left[ \frac{1}{(x-z)(x-z^*)[x-(1/z)][x-(1/z^*)]} \right] \right\}_{x=0} \quad (A.12) \]

\[ z = re^{i\phi}. \quad (A.13) \]

These equations were used to produce the plots in Section 4.4.3. The correct form of A.8 was given in [75].
As described in Section 4.3.3.1, a servo’s loop filter is used to shape the magnitude and phase response of the loop transfer function. The transfer function of the loop filter $G(s)$ is a complex function of the Laplace variable $s = j\omega$. The amplitude response of the filter is $|G(s)|$ and its phase response is $\arg\{G(s)\}$.

A symbolic representation of a loop filter based on an operational amplifier is shown in Figure B.1. Amplifiers of this type are treated in many electronics textbooks such as [76]. The filter’s transfer function is the ratio of the complex impedances $Z_1(s)$ and $Z_2(s)$ [49] given as:

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

$$= -\frac{Z_2(s)}{Z_1(s)}.$$  

Figure B.1: Model for an Op-Amp based loop filter
A schematic representing the loop filter used for the cooling laser “Slow Servo” (Section 4.3.3.3) is shown in Figure B.2. The transfer function for this servo when the switch is closed is:

\[ G(s) = -\frac{R_2}{R_1} \frac{1 + sR_3C}{[1 + s(R_2 + R_3)C]} \]  \hspace{1cm} (B.3)

This transfer function has a pole, defined as the frequency where \( G(s) \rightarrow \infty \) at:

\[ s = -\frac{1}{(R_2 + R_3)C} \]  \hspace{1cm} (B.4)

and a zero where \( G(s) \rightarrow 0 \) at:

\[ s = -\frac{1}{R_3C} \]  \hspace{1cm} (B.5)

The effect of the pole is to cause \(|G(s)|\) to roll off at 6dB / octave with a phase shift of \( \arg\{G(s)\} = -90^\circ \) beginning near the frequency of the pole. A zero has the opposite effect causing \(|G(s)|\) to increase at 6dB / octave and the phase to advance by \( \arg\{G(s)\} = 90^\circ \). [49]. For the case of the slow servo, this zero has the effect of canceling the lower frequency pole causing the filter response to flatten out starting at frequencies close to the frequency of the zero.
As an example a Bode plot of the amplitude and phase response of the loop filter used for the “Slow PZT Servo” is given in Figure B.3. The parameters used for this filter are $R_1 = 100\, \text{k}\Omega$, $R_2 = 1\, \text{M}\Omega$, $R_3 = 8.25\, \text{k}\Omega$ and $C = 0.01\, \mu\text{F}$. This gives a pole at $f_p = 0.016\, \text{Hz}$ and a zero at $f_z = 1.9\, \text{kHz}$ when the switch is closed. When the switch is opened, $R_2 \rightarrow \infty$, and the transfer function behaves like an integrator with infinite gain at DC.

![Bode plot](image)

**Figure B.3:** Amplitude Response (top) and phase response (bottom) of $G(s)$ for slow servo with switch closed. $R_1 = 100\, \text{k}\Omega$, $R_2 = 1\, \text{M}\Omega$, $R_3 = 8.25\, \text{k}\Omega$ and $C = 0.01\, \mu\text{F}$
C  Servo Circuits

Electronic circuits for laser line narrowing onto a Fabry-Perot resonance as well as stabilization onto a saturated absorption feature in Rb. The following circuits have been included:

- Circuit for low noise photodetector p. 120
- Circuit for slow PZT servo p. 121
- Circuit for DL100 PZT summing amp and current feedforward p. 122
- Circuit for DL100 current shunt p. 124
- Circuit for fast current control (CC) servo p. 123
- Circuit for 3f lock to Rb line p. 125
Figure C.1: Low Noise >1MHz BW Photo Detector for FP Lock
Figure C.2: Circuit for Slow Servo
Figure C.3: Summing amp and current feedforward
Figure C.4: Current Control Servo Circuit
Figure C.5: Current Shunt Circuit for DL100 Laser
Figure C.6: Circuit for 3f Lock Servo